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Problem 1 (25 pts)

A spline fit through the points (1,0), (2,4), and (3,9) is required. Find the coefficients b_1, c_1, a_2, b_2, and c_2 in the function

$$f(x) = \begin{cases} b_1x + c_1 & 1 \le x \le 2 \\ a_2x^2 + b_2x + c_2 & 2 \le x \le 3 \end{cases}$$

At $x = 1$, $b_1 + c_1 = 0$

$x = 3$, $9a_2 + 3b_2 + c_2 = 9$

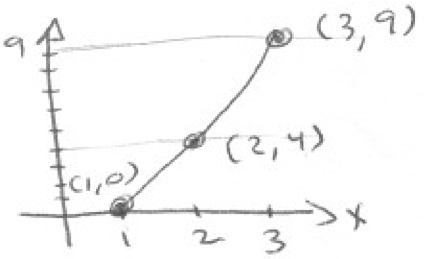
At $x = 2$, $2b_1 + c_1 = 4$

$4a_2 + 2b_2 + c_2 = 4$

$b_1 + c_1 = 0$
$2b_1 + c_1 = 4$

$b_1 = 4$
$c_1 = -4$

Slopes are equal: $b_1 = 2a_2(2) + b_2$



$9a_2 + 3b_2 + c_2 = 9$
$4a_2 + 2b_2 + c_2 = 4$
$4a_2 + b_2 = 4$

$$\begin{pmatrix} 1 & \frac{1}{4} & 0 & 1 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 9 \end{pmatrix} \sim \begin{pmatrix} 1 & \frac{1}{4} & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & \frac{3}{4} & 1 & 0 \end{pmatrix} \sim \begin{array}{c} \begin{array}{cccc} a_2 & b_2 & c_2 & \end{array} \\ \begin{pmatrix} 1 & \frac{1}{4} & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \end{pmatrix} \end{array}$$

$\Rightarrow \frac{1}{4}c_2 = 0$, $c_2 = 0$

$b_2 = 0$

$a_2 = 1$

Ans. $b_1 = 4$

$c_1 = -4$

$a_2 = 1$

$b_2 = 0$

$c_2 = 0$

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Problem 2 (25 pts)

Find the equation of the Newton Divided Difference polynomial which passes through the points (-1,0), (0,2), (1,0) and (2,0). Assume $x_0=-1$, $x_1=0$, $x_2=1$, and $x_3=2$. Show that the polynomial reduces to the product of the three linear factors (x+1)(x-1)(x-2) since the roots are located at x=-1,1 and 2.

i	x_i	$f(x_i)$ (b_0)	Δ (b_1)	Δ^2 (b_2)	Δ^3 (b_3)
0	-1	0	2	-2	1
1	0	2	-2	1	
2	1	0	0		
3	2	0			

$$f[x_1,x_0] = \frac{f(x_1)-f(x_0)}{x_1-x_0} = \frac{2-0}{0-(-1)} = 2$$

$$f[x_2,x_1] = \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{0-2}{1-0} = -2$$

$$f[x_3,x_2] = \frac{f(x_3)-f(x_2)}{x_3-x_2} = \frac{0-0}{2-1} = 0$$

$$f[x_2,x_1,x_0] = \frac{f[x_2,x_1]-f[x_1,x_0]}{x_2-x_0} = \frac{-2-2}{1-(-1)} = -2$$

$$f[x_3,x_2,x_1] = \frac{f[x_3,x_2]-f[x_2,x_1]}{x_3-x_1} = \frac{0-(-2)}{2-0} = 1$$

$$f[x_3,x_2,x_1,x_0] = \frac{f[x_3,x_2,x_1]-f[x_2,x_1,x_0]}{x_3-x_0} = \frac{1-(-2)}{2-(-1)} = 1$$

Ans. $f_3(x) =$ $b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$

$= 2(x+1) - 2(x+1)x + (x+1)x(x-1)$

$= (x+1)[2-2x+x^2-x] = (x+1)(x^2-3x+2)$

$= (x+1)(x-1)(x-2)$

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Problem 3 (25 pts)

The exponential $f(x) = e^x$ is to be approximated by a linear function. Evaluate the exponential at x=0,1,2,3 and find the equation of the least squares line based on the four data points generated. Find SSE, SSR, SST and the coefficient of determination r^2. Prepare a table to show how the coefficients of the normal equations were obtained. Prepare a second table to show how SSE, SSR, and SST were calculated.

x_i	Y_i	x_i^2	x_iY_i
0	1	0	0
1	2.718	1	2.718
2	7.389	4	14.778
3	20.086	9	60.258

$\Sigma x = 6$ $\Sigma Y = 31.193$ $\Sigma x^2 = 14$ $\Sigma xy = 77.754$

$\bar{Y} = 7.798$

$\hat{Y} = a_0 + a_1 x$

$4a_0 + 6a_1 = 31.193$

$6a_0 + 14a_1 = 77.754$

$$a_0 = \frac{\begin{vmatrix} 31.193 & 6 \\ 77.754 & 14 \end{vmatrix}}{\begin{vmatrix} 4 & 6 \\ 6 & 14 \end{vmatrix}} = -1.491$$

$a_1 = 6.193$

x_i	Y_i	$(Y_i - \bar{Y})^2$	$\hat{Y}_i$	$(Y_i - \hat{Y}_i)^2$
0	1	46.213	-1.491	6.205
1	2.718	25.806	4.702	3.936
2	7.389	0.167	10.895	12.292
3	20.086	150.995	17.088	8.988

SST = 223.181 SSE = 31.421

$\hat{Y} = -1.491 + 6.193X$

$SSR = SST - SSE$

$= 223.181 - 31.421$

$= 191.760$

$$r^2 = \frac{SSR}{SST} = \frac{191.760}{223.181} = 0.859$$

Ans. SSE = 31.421, **SSR =** 191.760, **SST =** 223.181, r^2 **=** 0.859

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Problem 4 (25 pts)

Find the equation of the Lagrange interpolating polynomial through the points in the table below. Estimate the value of the function when x = 2.

x	y
0	0
1	1
3	27
5	125

$$f_3(x) = \sum_{i=0}^{3} L_i(x) f(x_i) = L_1(x) + 27 L_2(x) + 125 L_3(x)$$

$$L_1(x) = \frac{x(x-3)(x-5)}{(1-0)(1-3)(1-5)} = \frac{x(x-3)(x-5)}{8}$$

$$L_2(x) = \frac{x(x-1)(x-5)}{(3-0)(3-1)(3-5)} = \frac{-x(x-1)(x-5)}{12}$$

$$L_3(x) = \frac{x(x-1)(x-3)}{(5-0)(5-1)(5-3)} = \frac{x(x-1)(x-3)}{40}$$

$$f_3(2) = \frac{1}{8} \cdot 2(-1)(-3) - \frac{27}{12} \cdot 2(1)(-3) + \frac{25}{8} \cdot 2(1)(-1)$$

$$= \frac{6}{8} + \frac{27}{2} - \frac{25}{4} = 8$$

....

Ans. $f_3(x) = \frac{1}{8} x(x-3)(x-5) - \frac{27}{12} x(x-1)(x-5) + \frac{25}{8} x(x-1)(x-3)$

$f_3(2) = 8$

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Problem 2 (25 pts)

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....

Ans. $f_3(x)$ =

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Problem 3 (25 pts)

The exponential $f(x) = e^x$ is to be approximated by a linear function. Evaluate the exponential at x=0,1,2,3 and find the equation of the least squares line based on the four data points generated. Find SSE, SSR, SST and the coefficient of determination r^2. Prepare a table to show how the coefficients of the normal equations were obtained. Prepare a second table to show how SSE, SSR, and SST were calculated. Use three places after the decimal point in <u>all</u> intermediate calculations.

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Problem 4 (25 pts)

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x	y
0	0
1	1
3	27
5	125

....

Ans. $f_3(x) =$

$f_3(2) =$