Su 94
EGN 3420 EXAM 1 Name Solutions

Problem 1 (25 pts)

Consider the function $f(x) = x^{1/3} - 2$.

A) Use the Bisection Method to estimate the root which is located between x=0 and x=10. Continue the iterations until the approximate relative error (in magnitude) is less than 10%. Fill in the table below. Enter all calculations rounded to the 4th place after the decimal point.

x_l	x_u	x_r	$\|e_A\|$,%
0	10	5	-
5	10	7.5	33.3333
7.5	10	8.75	14.2857
7.5	8.75	8.125	7.6923

B) Use the Newton-Raphson Method to estimate the root of the same function, starting at x=1. Continue until the true error (in magnitude) falls below 0.5. Fill in the table below. Enter all calculations rounded to the 4th place after the decimal point.

i	x_i	$f(x_i)$	$f'(x_i)$	E_T
0	1	-1	0.3333	
1	4			
2	7.1191			
3	7.9660			

A) $f(x) = x^{1/3} - 2$

$x_L = 0, x_u = 10 \Rightarrow x_r = \dfrac{x_L + x_u}{2} = \dfrac{0+10}{2} = 5$

$f(x_L) = f(0) = -2$

$f(x_r) = f(5) = 5^{1/3} - 2 = -0.2900$

$f(x_L) f(x_r) = (-2)(-0.2900) > 0$

$x_L = 5, x_u = 10 \Rightarrow x_r = \dfrac{5+10}{2} = 7.5$

$e_A = \left| \dfrac{x_r^{new} - x_r^{old}}{x_r^{new}} \right| \times 100 = \left| \dfrac{7.5 - 5}{7.5} \right| \times 100 = 33.3333\%$

$f(x_r) = 7.5^{1/3} - 2 = -0.0426$

$f(x_L) f(x_r) = (-0.2900)(-0.0426) > 0$

$x_L = 7.5, x_u = 10 \Rightarrow x_r = \dfrac{7.5+10}{2} = 8.75$

$e_A = \left| \dfrac{8.75 - 7.5}{8.75} \right| \times 100 = 14.2857\%$

$f(x_r) = f(8.75) = 8.75^{1/3} - 2 = 0.0606$

$f(x_L) f(x_r) = (-0.0426)(0.0606) < 0$

$x_L = 7.5, x_u = 8.75 \Rightarrow x_r = \dfrac{7.5 + 8.75}{2} = 8.125$

$e_A = \left| \dfrac{8.125 - 8.75}{8.125} \right| \times 100 = 7.6923\%$

B) $f(x) = x^{1/3} - 2$

$f'(x) = \frac{1}{3} x^{-2/3}$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} = x_i - \frac{x_i^{1/3} - 2}{\frac{1}{3} x_i^{-2/3}}$$

$$= x_i - 3x_i^{2/3}(x_i^{1/3} - 2)$$

$$= x_i - 3(x_i - 2x_i^{2/3})$$

$$= -2x_i + 6x_i^{2/3}$$

$$= 2(3x_i^{2/3} - x_i)$$

$\Rightarrow x_1 = 2(3x_0^{2/3} - x_0)$, $x_0 = 1$

$= 2(3-1)$

$= 4$

$x_2 = 2(3x_1^{2/3} - x_1)$, $x_1 = 4$

$= 2[3(4)^{2/3} - 4]$

$= 7.1191$

$x_3 = 2(3x_2^{2/3} - x_2)$, $x_2 = 7.1191$

$= 2[3(7.1191)^{2/3} - 7.1191]$

$= 7.9660$

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Problem 2 (25 pts)

Consider the function $f(x) = e^{2x}$. (Round all calculations to 4 places after the decimal point.)

A) Estimate f(0.5) using the 1st, 2nd, 3rd and 4th order truncated Taylor Series Approximation expanded about $x=0$.

Ans. 2 **(1st)** 2.5 **(2nd)** 2.6667 **(3rd)** 2.7083 **(4th)**

B) Calculate the approximate relative error in going from an estimate based on the 3rd order truncated series expansion to the estimate based on the 4th order truncated series expansion.

Ans. $e_A =$ 0.0154

C) Calculate the true relative error in the estimate based on the 4th order truncated series expansion.

Ans. $e_T =$ 0.0037

Work Area

A) $f(x) = e^{2x}$

$$f_1(x) = f(x_0) + f'(x_0)(x-x_0)$$

$$= e^{2x_0} + 2e^{2x_0}(x-x_0)$$

$$f_1(0.5) = e^0 + 2e^0(0.5-0) = 1 + 2(0.5) = 2$$

$$f_2(x) = f_1(x) + \frac{f''(x_0)}{2!}(x-x_0)^2$$

$$= f_1(x) + \frac{4e^{2x_0}}{2}(x-x_0)^2$$

$$f_2(0.5) = f_1(0.5) + 2e^0(0.5-0)^2$$

$$= 2 + 2(0.25) = 2.5$$

$$f_3(x) = f_2(x) + \frac{f'''(x_0)}{3!}(x-x_0)^3$$

$$= f_2(x) + \frac{8e^{2x_0}}{6}(x-x_0)^3$$

$$f_3(0.5) = f_2(0.5) + \frac{4}{3}e^0(0.5-0)^3$$

$$= 2.5 + \frac{4}{3}(0.125) = 2.6667$$

$$f_4(x) = f_3(x) + \frac{f^{iv}(x_0)}{4!}(x-x_0)^4$$

$$= f_3(x) + \frac{16e^{2x_0}}{24}(x-x_0)^4$$

$$f_4(0.5) = f_3(0.5) + \frac{2}{3}e^0(0.5-0)^4$$

$$= 2.6667 + \frac{2}{3}(0.0625) = 2.7083$$

B) $$e_A = \frac{2.7083 - 2.6667}{2.7083} = 0.0154$$

c) $$f(0.5) = e^{2(0.5)}$$

$$= e$$

$$e_T = \frac{e - 2.7083}{e} = 0.0037$$

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Problem 3 (25 pts)

Solve the following system of equations using the inverse matrix in the solution.

$$\begin{aligned} x &+ y + z = 6 \\ 2x &- y - z = -3 \\ 5x &+ 4y - 2z = 7 \end{aligned}$$

Ans. $A^{-1} = \frac{1}{18}\begin{pmatrix} 6 & 6 & 0 \\ -1 & -7 & 3 \\ 13 & 1 & -3 \end{pmatrix}$, x = $\underline{\ 1\ }$, y = $\underline{\ 2\ }$, z = $\underline{\ 3\ }$

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Work Area

$A\underline{x} = \underline{b}$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 5 & 4 & -2 \end{pmatrix}, \quad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -1 \\ 5 & 4 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & -3 \\ 0 & -1 & -7 \end{vmatrix} = 21 - 3 = 18$$

$$A^c = \begin{pmatrix} 6 & -1 & 13 \\ 6 & -7 & 1 \\ 0 & 3 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \operatorname{Adj} A = \frac{1}{18}\begin{pmatrix} 6 & 6 & 0 \\ -1 & -7 & 3 \\ 13 & 1 & -3 \end{pmatrix}$$

$$\underline{x} = A^{-1}\underline{b}$$

$$= \frac{1}{18}\begin{pmatrix} 6 & 6 & 0 \\ -1 & -7 & 3 \\ 13 & 1 & -3 \end{pmatrix}\begin{pmatrix} 6 \\ -3 \\ 7 \end{pmatrix}$$

$$= \frac{1}{18}\begin{pmatrix} 18 \\ 36 \\ 54 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

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Problem 4 (25 pts)

Solve the following system of equations by Gauss Elimination. Start with the augmented matrix and perform a series of elementary row operations on the matrix until the forward phase of the Gauss Elimination is complete. Finish the solution using back substitution.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 3 \\ 2x_1 \qquad\quad - 9x_3 &= -1 \\ x_1 + 6x_2 + 6x_3 &= 6 \end{aligned}$$

$$(A \mid \underline{b}) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 2 & 0 & -9 & -1 \\ 1 & 6 & 6 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & -4 & -15 & -7 \\ 0 & 4 & 3 & 3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 15/4 & 7/4 \\ 0 & 4 & 3 & 3 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 15/4 & 7/4 \\ 0 & 0 & -12 & -4 \end{array}\right)$$

$$\sim \begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left(\begin{array}{ccc|c} 1 & 2 & 3 & 3 \\ 0 & 1 & 15/4 & 7/4 \\ 0 & 0 & 1 & 1/3 \end{array}\right) \end{array}$$

$x_3 = \frac{1}{3}$

$x_2 + \frac{15}{4} x_3 = \frac{7}{4}$

$x_2 = \frac{7}{4} - \frac{15}{4}\left(\frac{1}{3}\right) = \frac{1}{2}$

$x_1 + 2x_2 + 3x_3 = 3$

$x_1 = 3 - 2(1/2) - 3(1/3) = 1$

Soln. $x_1 = 1$

$x_2 = \frac{1}{2}$

$x_3 = \frac{1}{3}$

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x_l	x_u	x_r	$\lvert e_A \rvert$,%
0	10		

B) Use the Newton-Raphson Method to estimate the root of the same function, starting at $x=1$. Continue until the true error (in magnitude) falls below 0.5. Fill in the table below. Enter all calculations rounded to the 4th place after the decimal point.

i	x_i	$f(x_i)$	$f'(x_i)$	E_T
0	1			

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Problem 2 (25 pts)

Consider the function $f(x) = e^{2x}$. (Round all calculations to 4 places after the decimal point.)

A) Estimate f(0.5) using the 1st, 2nd, 3rd and 4th order truncated Taylor Series Approximation expanded about x=0.

Ans. _______ (1st) _______ (2nd) _______ (3rd) _______ (4th)

B) Calculate the approximate relative error in going from an estimate based on the 3rd order truncated series expansion to the estimate based on the 4th order truncated series expansion.

Ans. e_A = __________

C) Calculate the true relative error in the estimate based on the 4th order truncated series expansion.

Ans. e_T = __________

....

Work Area

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Problem 3 (25 pts)

Solve the following system of equations using the inverse matrix in the solution.

$$
\begin{aligned}
x + y + z &= 6 \\
2x - y - z &= -3 \\
5x + 4y - 2z &= 7
\end{aligned}
$$

Ans. $A^{-1} = \left(\quad \right)$, $x =$ ____, $y =$ ____, $z =$ ____

....

Work Area

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Problem 4 (25 pts)

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$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 3 \\ 2x_1 \qquad\quad - 9x_3 &= -1 \\ x_1 + 6x_2 + 6x_3 &= 6 \end{aligned}$$

Ans. x_1 = _____, x_2 = _____, x_3 = _____

Work Area