Exam 1

SP 97

EGN 3420 ~~QUIZ~~ Name_____

3)

Consider the following system of equations:

$$
\begin{array}{rrrrrrrrrrr}
 & & v & + & w & + & x & + & 2y & = & 4 \\
u & + & v & + & w & + & x & - & y - z & = & 2 \\
2u & & & - & 3w & + & x & & + z & = & 3 \\
-u & & & + & 2w & + & x & + & y - z & = & 1 \\
u & - & v & & & & & & - 2z & = & 0
\end{array}
$$

A) Transform the augmented matrix (A|b) into its Echelon Form by performing a sequence of elementary row operations. Without solving for the general solution, determine if z can be chosen arbitrarily. Repeat for x and y.

B) Start with the Echelon Form and use back substitution to find the general solution expressed in terms of one or more abitrary unknowns.

$$
\left[\begin{array}{cccccc:c}
1 & 1 & 1 & 1 & -1 & -1 & 2 \\
0 & 1 & 1 & 1 & 2 & 0 & 4 \\
2 & 0 & -3 & 1 & 0 & 1 & 3 \\
-1 & 0 & 2 & 1 & 1 & -1 & 1 \\
1 & -1 & 0 & 0 & 0 & -2 & 0
\end{array}\right]
\qquad
\left[\begin{array}{cccccc:c}
 & & & & & & \\
 & & & & & & \\
 & & & & & & \\
 & & & & & & \\
 & & & & & &
\end{array}\right]
$$

Original Augmented Matrix **Echelon Form**

z (is), (is not) arbitrary because

x (is), (is not) arbitrary because

y (is), (is not) arbitrary because

solution:

u =
v =
w =
x =
y =
z =

Quiz solns

$$
\begin{aligned}
v + w + x + 2y &= 4 \\
u + v + w + x - y - z &= 2 \\
2u - 3w + x + z &= 3 \\
-u + 2w + x + y - z &= 1 \\
u - v - 2z &= 0
\end{aligned}
$$

$$
\begin{bmatrix}
1 & 1 & 1 & 1 & -1 & -1 & 2 \\
0 & 1 & 1 & 1 & 2 & 0 & 4 \\
2 & 0 & -3 & 1 & 0 & 1 & 3 \\
-1 & 0 & 2 & 1 & 1 & -1 & 1 \\
1 & -1 & 0 & 0 & 0 & -2 & 0
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 1 & 1 & -1 & -1 & 2 \\
0 & 1 & 1 & 1 & 2 & 0 & 4 \\
0 & -2 & -5 & -1 & 2 & 3 & -1 \\
0 & 1 & 3 & 2 & 0 & -2 & 3 \\
0 & -2 & -1 & -1 & 1 & -1 & -2
\end{bmatrix}
$$

$$
\sim
\begin{bmatrix}
1 & 1 & 1 & 1 & -1 & -1 & 2 \\
0 & 1 & 1 & 1 & 2 & 0 & 4 \\
0 & 0 & -3 & 1 & 6 & 3 & 7 \\
0 & 0 & 2 & 1 & -2 & -2 & -1 \\
0 & 0 & 1 & 1 & 5 & -1 & 6
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 1 & 1 & -1 & -1 & 2 \\
0 & 1 & 1 & 1 & 2 & 0 & 4 \\
0 & 0 & 1 & 1 & 5 & -1 & 6 \\
0 & 0 & 2 & 1 & -2 & -2 & -1 \\
0 & 0 & -3 & 1 & 6 & 3 & 7
\end{bmatrix}
$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & 2 \\ 0 & 1 & 1 & 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 & 5 & -1 & 6 \\ 0 & 0 & 0 & -1 & -12 & 0 & -13 \\ 0 & 0 & 0 & 4 & 21 & 0 & 25 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & 2 \\ 0 & 1 & 1 & 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 & 5 & -1 & 6 \\ 0 & 0 & 0 & 1 & 12 & 0 & 13 \\ 0 & 0 & 0 & 0 & -27 & 0 & -27 \end{bmatrix}$$

$$\begin{matrix} u & v & w & x & y & z \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 & -1 & -1 & 2 \\ 0 & 1 & 1 & 1 & 2 & 0 & 4 \\ 0 & 0 & 1 & 1 & 5 & -1 & 6 \\ 0 & 0 & 0 & 1 & 12 & 0 & 13 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

z can be arbitrary since the 1st five colums make up a nonsingular matrix

i.e. $$\begin{vmatrix} 1 & 1 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 1 & 12 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = 1$$

To see if y is arbitrary we check the following determinant

$$\begin{vmatrix} 1 & 1 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix} = 0 \quad \therefore y \text{ is not arbitrary}$$

To see if x is arbitrary, check the determinant

$$\begin{vmatrix} 1 & 1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{vmatrix} = 0 \quad \therefore x \text{ is not arbitrary}$$

General soln.

$y = 1$

$x + 12y = 13 \Rightarrow x = 1$

$w + x + 5y - z = 6 \Rightarrow w = z$

$v + w + x + 2y = 4 \Rightarrow v = 1 - z$

$u + v + w + x - z = 2 \Rightarrow u = 1 + z$

$u + v = 2$

$u + 1 - z = 2$

$u = 1 + z$