For the system of equations below:

$$
\begin{array}{rcrcrcrcl}
w & + & x & & & + & z & = & 2 \\
3w & - & x & + & 3y & & & = & 3 \\
 & & x & - & 4y & + & 4z & = & 4 \\
w & - & x & + & 2y & - & z & = & 0
\end{array}
$$

1. Find the Echelon form of the augmented matrix. Are there any arbitrary unknowns? If so, how many arbitrary unknowns are there?
2. Without solving for x, determine if x can be arbitrary. If x can be arbitrary, find the solution for w, y and z in terms of x.
3. Without solving for y, determine if y can be arbitrary. If y can be arbitrary, find the solution for w, x and z in terms of y.

....
SHOW ALL WORK!

1.
$$
\left(\begin{array}{cccc|c}
1 & 1 & 0 & 1 & 2 \\
3 & -1 & 3 & 0 & 3 \\
0 & 1 & -4 & 4 & 4 \\
1 & -1 & 2 & -1 & 0
\end{array}\right)
\sim
\left(\begin{array}{cccc|c}
1 & 1 & 0 & 1 & 2 \\
0 & -4 & 3 & -3 & -3 \\
0 & 1 & -4 & 4 & 4 \\
0 & -2 & 2 & -2 & -2
\end{array}\right)
\sim
\left(\begin{array}{cccc|c}
1 & 1 & 0 & 1 & 2 \\
0 & 1 & -1 & 1 & 1 \\
0 & 1 & -4 & 4 & 4 \\
0 & -4 & 3 & -3 & -3
\end{array}\right)
$$

$$
\sim
\left(\begin{array}{cccc|c}
1 & 1 & 0 & 1 & 2 \\
0 & 1 & -1 & 1 & 1 \\
0 & 0 & -3 & 3 & 3 \\
0 & 0 & -1 & 1 & 1
\end{array}\right)
\sim
\left(\begin{array}{cccc|c}
1 & 1 & 0 & 1 & 2 \\
0 & 1 & -1 & 1 & 1 \\
0 & 0 & 1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

System reduces to 3 equations in 4 unknowns ⇒ 1 arbitrary unknown

2. Echelon Form
$$
\begin{array}{cccc}
w & x & y & z
\end{array}
$$
$$
\left(\begin{array}{cccc|c}
1 & 1 & 0 & 1 & 2 \\
0 & 1 & -1 & 1 & 1 \\
0 & 0 & 1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0
\end{array}\right)
$$

x is arbitrary if
$$
\begin{vmatrix}
1 & 0 & 1 \\
0 & -1 & 1 \\
0 & 1 & -1
\end{vmatrix}
\neq 0
$$

Since the above determinant is zero, x cannot be arbitrary.

3. Y is arbitrary if $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{vmatrix} \neq 0$

Since the above determinant is not zero, Y is arbitrary.

From the Echelon form,

$$
\begin{array}{ccc} w & x & z \end{array}
$$
$$
\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1+Y \\ 0 & 0 & -1 & -1-Y \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 1-Y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1+Y \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1-Y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1+Y \end{array} \right)
$$
$$
\begin{array}{ccc} w & x & z \end{array}
$$

Soln: $w = 1-Y$
        $x = 0$
        $z = 1+Y$
        $Y = $ arbitrary