EGN 3420 EXAM I Sp 96

Problem 1 (50 pts)

The function $f(x) = x^4 - 100$ has a root located between 0 and 5. Fill in the tables below for the first three iterations of the Bisection Method and the False Position method. Express all answers rounded to four digits after the decimal point.

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	$\lvert e_A \rvert$, %
1	2.0000	5.0000	3.5000	-84.0000	50.0625	—
2	2.0000	3.5000	2.7500	-84.0000	-42.8086	27.2727
3	2.7500	3.5000	3.1250	-42.8086	-4.6326	12.0000

Bisection Method

$$x_r = \frac{x_l + x_u}{2}$$

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_u)$	$f(x_r)$	$\lvert e_A \rvert$, %
1	2.0000	5.0000	2.4138	-84.0000	525.0000	-66.0531	—
2	2.4138	5.0000	2.7028	-66.0531	525.0000	-46.6335	10.6926
3	2.7028	5.0000	2.8902	-46.6335	525.0000	-30.2227	6.4840

False Position Method

$$x_r = x_l - f(x_l)\left\{\frac{x_u - x_l}{f(x_u) - f(x_l)}\right\}$$

Problem 2 (50 pts)

Consider the function $f(x) = x^3 + x - 10$.

A) Use the Simple One Point Iteration Method with $g(x) = (10-x)/x^2$ and fill in the table below. Stop when $|e_A| < 1\%$ or $i>3$ whichever occurs first. In the column for E_T, R is the true root of $f(x) = 0$ which you should be able to determine. Round all answers to 4 places after the decimal point for both Parts A) and B).

i	x_i	$E_T = R-x_i$	$\|e_A\|$, %
0	1.9500	0.0500	—
1	2.1170	-0.1170	7.8885
2	1.7589	0.2411	20.3593
3	2.6639	-0.6639	33.9727

$$x_{i+1} = g(x_i) = \frac{10 - x_i}{x_i^2}$$

B) Use the Newton-Raphson Method and fill in the table below.

i	x_i	$f(x_i)$	$f'(x_i)$
0	0.0000	-10.0000	1.0000
1	10.0000	1000.0000	301.0000
2	6.6777	294.4530	134.7767

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \qquad f(x_i) = x_i^3 + x_i - 10$$

$$f'(x_i) = 3x_i^2 + 1$$