Sp 95
EGN 3420

Exam 2 B

Name_______________

SHOW ALL WORK!

Problem 1 (25 pts)

Solve the following system of equations using the Gauss Jordan Elimination Method.

$$
\begin{array}{rcrcrcrcr}
a & + & b & + & c & + & d & = & 5 \\
2a & - & b & - & c & + & d & = & 25 \\
a & + & 3b & + & 2c & - & 4d & = & 0 \\
 & & 2b & - & 2c & + & d & = & 10
\end{array}
$$

Work Area

$$
(A|\underline{b}) = \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 2 & -1 & -1 & 1 & 25 \\ 1 & 3 & 2 & -4 & 0 \\ 0 & 2 & -2 & 1 & 10 \end{array}\right] \approx
$$

Ans. a = _____, b = _____, c = _____, d = _____

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Problem 2 (25 pts)

Find the equation of the Least Squares line which best fits the hyperbola $y = 1/x$ over the interval (1,5). Do this by generating 5 equally spaced points over the interval (1,5) from the hyperbola. Find the sum of the squares of the residuals SSE. Round all calculations to 4 places after the decimal point.

Work Area

i	x_i	$y_i = 1/x_i$
1	1	1.0000
2	2	
3	3	
4	4	
5	5	

Ans. $\hat{y} = a_0 + a_1 x$ where $a_0 =$ _____, $a_1 =$ _____

SSE = _________

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Problem 3 (25pts)

Consider the definite integral $I = \int_1^6 \frac{a}{x^2}\,dx$. Using Trapezoidal integration with a step size $\Delta x = 1$ results in an approximate value $I_T = 2.44375$.

A) Find the value of "a".
B) Using the correct value of "a", apply Simpson's 1/3 Formula with $\Delta x = 5/6$ and find I_S, the approximation to the definite integral.
C) Find the true relative error in Parts A) and B).

Work Area

i	x_i	$f_i = \frac{a}{x_i^2}$
0		
1		
2		
3		
4		
5		

Ans. A) a = _____ B) I_S = _________ C) e_T (%) = _________ Part A)

e_T(%) = _________ Part B)

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SHOW ALL WORK!

Problem 4 (25pts)

Using Trapezoidal integration to approximate the definite integral of $f(x) = \sin x$ from 0 to $\pi/4$, it is necessary to keep the truncation error below 2.854785×10^{-8}. Find the number of required sub-intervals from 0 to $\pi/4$.

....

....

Ans. $n =$ __________

Sp .95 EXAM 2B SOLNS

1

$$a + b + c + d = 5$$
$$2a - b - c + d = 25$$
$$a + 3b + 2c - 4d = 0$$
$$2b - 2c + d = 10$$

$$(A \mid \underline{b}) = \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 2 & -1 & -1 & 1 & 25 \\ 1 & 3 & 2 & -4 & 0 \\ 0 & 2 & -2 & 1 & 10 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & -3 & -3 & -1 & 15 \\ 0 & 2 & 1 & -5 & -5 \\ 0 & 2 & -2 & 1 & 10 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1/3 & -5 \\ 0 & 2 & 1 & -5 & -5 \\ 0 & 2 & -2 & 1 & 10 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1/3 & -5 \\ 0 & 0 & -1 & -17/3 & 5 \\ 0 & 0 & -4 & 1/3 & 20 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1/3 & -5 \\ 0 & 0 & 1 & 17/3 & -5 \\ 0 & 0 & 0 & 23 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1/3 & -5 \\ 0 & 0 & 1 & 17/3 & -5 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 5 \\ 0 & 1 & 1 & 0 & -5 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \sim \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \end{array}$$

Soln. $a = 10$

$b = 0$

$c = -5$

$d = 0$

② $Y = \frac{1}{X}$ $n = 5$

X_i	$Y_i = \frac{1}{X_i}$	$X_i Y_i$	X_i^2	$\hat{Y}_i = 1.0116 - 0.185 X_i$	$e_i = Y_i - \hat{Y}_i$
1	1	1	1	0.8266	0.1734
2	0.5	1	4	0.6416	-0.1416
3	0.3333	1	9	0.4566	-0.1233
4	0.25	1	16	0.2716	-0.0216
5	0.2	1	25	0.0866	0.1134

$\Sigma X_i = 15$ $\Sigma Y_i = 2.2833$ $\Sigma X_i Y_i = 5$ $\Sigma X_i^2 = 55$

$$n a_0 + (\Sigma X_i) a_1 = \Sigma Y_i$$

$$(\Sigma X_i) a_0 + (\Sigma X_i^2) a_1 = \Sigma X_i Y_i$$

$$5 a_0 + 15 a_1 = 2.2833$$

$$15 a_0 + 55 a_1 = 5$$

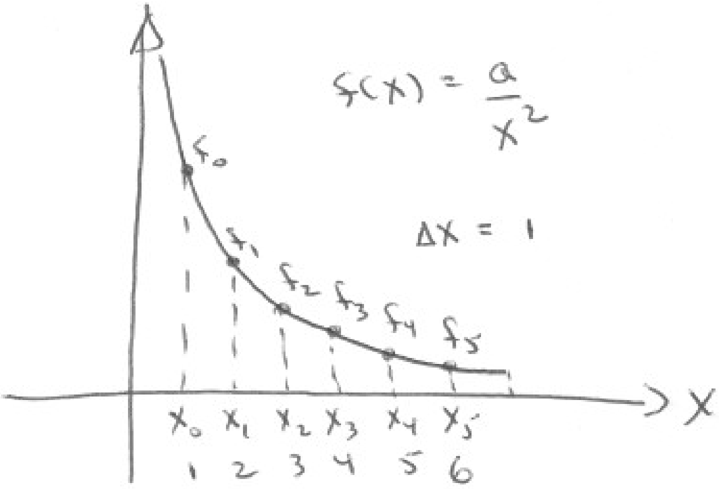
$$a_0 = \frac{\begin{vmatrix} 2.2833 & 15 \\ 5 & 55 \end{vmatrix}}{\begin{vmatrix} 5 & 15 \\ 15 & 55 \end{vmatrix}} = \frac{50.5815}{50} = 1.0116$$

$$a_1 = \frac{\begin{vmatrix} 5 & 2.2833 \\ 15 & 5 \end{vmatrix}}{50} = \frac{-9.2495}{50} = -0.1850$$

$$SSE = \Sigma e_i^2 = (0.1734)^2 + \dots + (0.1134)^2 = 0.0786$$

③

i	x_i	$f_i = a x_i^{-2}$
0	1	a
1	2	$a/4$
2	3	$a/9$
3	4	$a/16$
4	5	$a/25$
5	6	$a/36$



$$I_T = \Delta x\left[\frac{f_0+f_5}{2} + f_1+f_2+f_3+f_4\right]$$

$$= 1\left[\frac{a + a/36}{2} + \frac{a}{4} + \frac{a}{9} + \frac{a}{16} + \frac{a}{25}\right]$$

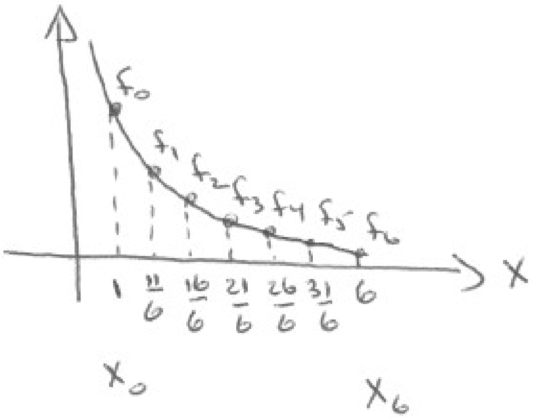
$$= \left[\frac{37}{72} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}\right] a$$

$$I_T = 0.9775a$$

$$2.44375 = 0.9775a$$

$$= 2.5$$

B) $f(x) = \frac{2.5}{x^2}$, $\Delta x = \frac{5}{6}$



i	x_i	$f_i = \frac{2.5}{x_i^2}$
0	1	2.5
1	11/6	0.7438
2	16/6	0.3516
3	21/6	0.2041
4	26/6	0.1331
5	31/6	0.0937
6	6	0.0694

$$I_s = \frac{\Delta x}{3}\left[f_0 + 4f_1 + 2f_2 + 4f_3 + 2f_4 + 4f_5 + f_6\right]$$

$$= \frac{5}{18}\left[2.5 + 4(0.7438 + 0.2041 + 0.0937) + 2(0.3516 + 0.1331) + 0.0694\right]$$

$$= 2.1403$$

c) $$I = \int_1^6 f(x)\,dx = \int_1^6 \frac{2.5}{x^2}\,dx = 2.5\frac{x^{-1}}{-1}\Big|_1^6 = \frac{-2.5}{x}\Big|_1^6$$

$$= \frac{-2.5}{6} + 2.5$$

$$= 2.0833$$

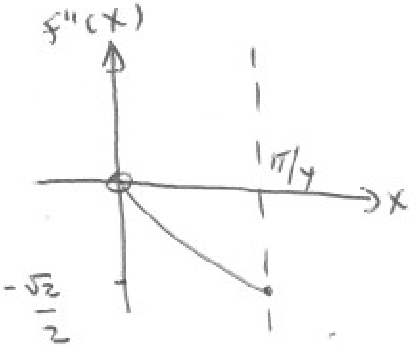
$$e_r(\%) = \frac{I - I_s}{I} \times 100$$

$$= \left(\frac{2.0833 - 2.1403}{2.0833}\right) \times 100 = -2.75\%$$

(4) $$I = \int_0^{\pi/4} \sin x \, dx \qquad f(x) = \sin x \qquad f''(x) = -\sin x$$

$$I - I_T = \frac{-1}{12} f''(\xi)(b-a)h^2$$

$$\text{Max} \, | I - I_T | = \frac{1}{12} \underset{0<x<\frac{\pi}{4}}{\text{Max}} |(-\sin x)| \, \frac{\pi}{4} h^2 = 2.854785 \times 10^{-8}$$



$$\frac{1}{12} \frac{\sqrt{2}}{2} \frac{\pi}{4} h^2 = 2.854185 \times 10^{-8}$$

$$h^2 = 61.68502835 \times 10^{-8}$$

$$h = 7.853981688 \times 10^{-4}$$

$$\frac{\pi/4}{n} = 7.853981688 \times 10^{-4}$$

$$n = \frac{\pi}{4(7.853981688)} \times 10^4$$

$$= 0.1 \times 10^4$$

$$n = 1000$$