Sp 95

EGN 3420 Exam 2A Name__________________

SHOW ALL WORK!

Problem 1 (40 pts)

Consider the following system of equations:

$$\begin{array}{rrrrrcr} x_1 & + x_2 & + x_3 & - 2x_4 & + 2x_5 & = & 1 \\ 2x_1 & - x_2 & + 3x_3 & - 2x_4 & + 2x_5 & = & 3 \\ 3x_1 & & - 4x_3 & + 4x_4 & - 4x_5 & = & -4 \\ 4x_1 & - 3x_2 & & + 3x_4 & - 3x_5 & = & 0 \\ x_1 & & + 3x_3 & - 3x_4 & + 3x_5 & = & 3 \end{array}$$

A) Transform the augmented matrix into Echelon Form.

Work Area

$$\left(A|\underline{b}\right) = \left[\begin{array}{rrrrr|r} 1 & 1 & 1 & -2 & 2 & 1 \\ 2 & -1 & 3 & -2 & 2 & 3 \\ 3 & 0 & -4 & 4 & -4 & -4 \\ 4 & -3 & 0 & 3 & -3 & 0 \\ 1 & 0 & 3 & -3 & 3 & 3 \end{array}\right] \approx$$

Check the appropriate box.

- ☐ The equations are inconsistent.
- ☐ The equations are consistent and there is a unique solution.
- ☐ The equations are consistent and there is 1 arbitrary unknown.
- ☐ The equations are consistent and there are 2 arbitrary unknowns.

B) If the equations are consistent and there is a unique solution, find it.
If the equations are consistent and there is 1 arbitrary unknown, determine if x_1 can be arbitrary <u>without solving for it</u>.
If the equations are consistent and there are 2 arbitrary unknowns, determine if x_1 can be one of the arbitrary unknowns <u>without solving for it</u>.

Work Area

C) If the equations are consistent and there are one or more arbitrary unknowns, express the solution in terms of the arbitrary unknown(s).

Ans. x_1 = ______ x_2 = ______ x_3 = ______ x_4 = ______ x_5 = ______

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Problem 2 (30 pts)

Find the equation of the Least Squares line which best fits the quadratic $y = x^2 + 1$ over the interval (0,5). Do this by generating 6 equally spaced points over the interval (0,5) from the quadratic.

Work Area

i	x_i	$y_i = x_i^2 + 1$
1	0	1.0000
2	1	
3	2	
4	3	
5	4	
6	5	

Ans. $\hat{y} = a_0 + a_1 x$ where $a_0 =$ _____ , $a_1 =$ _____

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Problem 3 (30 pts)

Consider the definite integral $I = \int_0^1 e^{-2x}\, dx$.

A) Use Trapezoidal integration with n=5 intervals to approximate I.
B) Find the truncation error.

Round all calculations to 5 places after the decimal point.

Work Area

i	x_i	$f_i = e^{-2x_i}$
0		
1		
2		
3		
4		
5		

Ans. A) I_T = ________ B) Truncation Error = ________

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EXAM 2A SOLNS

(1)

$$\begin{aligned}
x_1 + x_2 + x_3 - 2x_4 + 2x_5 &= 1 \\
2x_1 - x_2 + 3x_3 - 2x_4 + 2x_5 &= 3 \\
3x_1 \quad - 4x_3 + 4x_4 - 4x_5 &= -4 \\
4x_1 - 3x_2 \quad + 3x_4 - 3x_5 &= 0 \\
x_1 \quad + 3x_3 - 3x_4 + 3x_5 &= 3
\end{aligned}$$

A)

$$(A \vdots \underline{b}) = \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 2 & 1 \\ 2 & -1 & 3 & -2 & 2 & 3 \\ 3 & 0 & -4 & 4 & -4 & -4 \\ 4 & -3 & 0 & 3 & -3 & 0 \\ 1 & 0 & 3 & -3 & 3 & 3 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 2 & 1 \\ 0 & -3 & 1 & 2 & -2 & 1 \\ 0 & -3 & -7 & 10 & -10 & -7 \\ 0 & -7 & -4 & 11 & -11 & -4 \\ 0 & -1 & 2 & -1 & 1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 2 & 1 \\ 0 & 1 & -2 & 1 & -1 & -2 \\ 0 & -3 & -7 & 10 & -10 & -7 \\ 0 & -7 & -4 & 11 & -11 & -4 \\ 0 & -3 & 1 & 2 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 2 & 1 \\ 0 & 1 & -2 & 1 & -1 & -2 \\ 0 & 0 & -13 & 13 & -13 & -13 \\ 0 & 0 & -18 & 18 & -18 & -18 \\ 0 & 0 & -5 & 5 & -5 & -5 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & 1 & 1 & -2 & 2 & 1 \\ 0 & 1 & -2 & 1 & -1 & -2 \\ 0 & 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$
\begin{array}{ccccc|c}
x_1 & x_2 & x_3 & x_4 & x_5 & \\
\end{array}
$$

$$
\left[\begin{array}{ccccc|c}
1 & 1 & 1 & -2 & 2 & 1 \\
0 & 1 & -2 & 1 & -1 & -2 \\
0 & 0 & 1 & -1 & 1 & 1
\end{array}\right]
$$

The equations are consistent. There are 2 arbitrary unknowns.

B) x_4 & x_5 can be arbitrary since the 1st 3 columns of the Echelon Matrix form a non-singular matrix. Solving for x_1, x_2, x_3

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c) $x_3 = 1 + x_4 - x_5$

$x_2 = -2 + 2x_3 - x_4 + x_5$

$= -2 + 2(1 + x_4 - x_5) - x_4 + x_5$

$= x_4 - x_5$

$x_1 = 1 - x_2 - x_3 + 2x_4 - 2x_5$

$= 1 - (x_4 - x_5) - (1 + x_4 - x_5) + 2x_4 - 2x_5$

$= 0$

Soln.

$x_1 = 0$

$x_2 = x_4 - x_5$

$x_3 = 1 + x_4 - x_5$

x_4, x_5 arbitrary

Removing the x_1 and x_5 columns from the Echelon Matrix and checking the determinant of the remaining matrix,

$$\begin{vmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 1 & -1 \end{vmatrix} = \begin{vmatrix} -3 & 3 \\ 1 & -1 \end{vmatrix} = 0$$

Therefore x_1 & x_5 cannot both be arbitrary

Since we have shown that x_4 & x_5 can be arbitrary, this implies that x_1 is not arbitrary.

② $y = x^2 + 1$ $n = 6$

x_i	y_i	x_i^2	$x_i y_i$
0	1	0	0
1	2	1	2
2	5	4	10
3	10	9	30
4	17	16	68
5	26	25	130

$\Sigma x_i = 15$ $\Sigma y_i = 61$ $\Sigma x_i^2 = 55$ $\Sigma x_i y_i = 240$

$$n a_0 + (\Sigma x_i) a_1 = \Sigma y_i$$

$$(\Sigma x_i) a_0 + (\Sigma x_i^2) a_1 = \Sigma x_i y_i$$

$$6a_0 + 15a_1 = 61$$

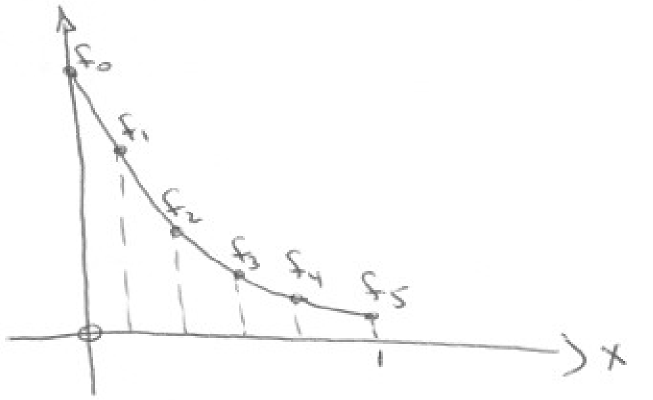
$$15a_0 + 55a_1 = 240$$

$$a_0 = \frac{\begin{vmatrix} 61 & 15 \\ 240 & 55 \end{vmatrix}}{\begin{vmatrix} 6 & 15 \\ 15 & 55 \end{vmatrix}} = \frac{-245}{105} = -2.333$$

$$a_1 = \frac{\begin{vmatrix} 6 & 61 \\ 15 & 240 \end{vmatrix}}{\begin{vmatrix} 6 & 15 \\ 15 & 55 \end{vmatrix}} = \frac{525}{105} = 5$$

(3) $I = \int_0^1 e^{-2x}\,dx$

A)



i	x_i	$f_i = e^{-2x_i}$
0	0	1
1	0.2	0.67032
2	0.4	0.44933
3	0.6	0.30119
4	0.8	0.20190
5	1	0.13534

$$I_T = \Delta x \left[\frac{f_0 + f_5}{2} + f_1 + f_2 + f_3 + f_4 \right]$$

$$= 0.2 \left[\frac{1 + 0.13534}{2} + 0.67032 + \cdots + 0.20190 \right]$$

$$= 0.43808$$

B) $I - I_T$ = Truncation Error

$$I = \int_0^1 e^{-2x}\,dx = \left. \frac{e^{-2x}}{-2} \right|_0^1 = \frac{1}{2}\left(1 - e^{-2}\right) = 0.43233$$

$$I - I_T = 0.43233 - 0.43808$$

$$= -0.00575$$