Sp 95 Exam 1B Name ________________

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SHOW ALL WORK!

Problem 1 (25 pts)

Consider the function $f(x) = 1/x$.

A) Find the 3rd order truncated Taylor Series Expansion of $f(x)$ about the pt $x_0 = 1/2$.

B) Use the 3rd order truncated series to estimate $f(x)$ when $x = 0.6$ to 3 places after the decimal point.

C) Find the true relative error (in %) in the answer to Part B) to 2 places after the decimal point..

....WORK AREA.......................................

....ANSWERS.......................................

A) $f_3(x) =$ ______________________________

B) $f_3(0.6) =$ ____________

C) $e_T =$ ________ %

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Problem 2 (25 pts)

Given the function $f(x) = x^3 - \frac{16}{x}$, use the Newton Raphson Method to fill in the table below. Round all calculations to 4 places after the decimal point, except the last column which should be rounded to 2 places after the decimal point. Stop when the magnitude of the approximate relative error is less than 1% or after 5 iterations, whichever comes first.

i	x_i	$f(x_i)$	$f'(x_i)$	$\lvert e_A \rvert$, %
0	1			----
1				
2				
3				
4				
5				

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Problem 3 (25 pts)

The following data points come from an unknown function f(x).

(0,1), (1,a), (2,5), (3,19)

A) Find the value of "a" if the coefficient b_3 in the Newton Divided Difference Interpolating polynomial $f_3(x)$ is 1.

B) Find $f_3(x)$.

The table below may help in your solution.

i	x_i	$f(x_i)$	Δ	Δ^2	Δ^3
0	0	1			
1	1	a			
2	2	5			
3	3	19			

....WORK AREA..

....ANSWERS..

A) a = ________ , B) $f_3(x)$ = ________________________________

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Problem 4 (25 pts)

A) Complete the table below to find the coefficients of the Newton Divided Difference 3rd order interpolating polynomial, i.e.

$$f_3(x) = b_0 + b_1(x\text{-}x_0) + b_2(x\text{-}x_0)(x\text{-}x_1) + b_3(x\text{-}x_0)(x\text{-}x_1)(x\text{-}x)$$

i	x_i	$f(x_i)$	Δ	Δ^2	Δ^3
0	0	-1			
1	2	1			
2	3	3.5			
3	4	7			

B) Use $f_3(x)$ to estimate $f(1)$.

....WORK AREA..

....ANSWERS..

b_0 = ________, b_1 = ________, b_2 = ________, b_3 = ________

$f_3(1)$ = ________

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EXAM 1 Solutions

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① $f(x) = \frac{1}{x}$, $f'(x) = \frac{-1}{x^2}$, $f''(x) = \frac{2}{x^3}$, $f'''(x) = \frac{-6}{x^4}$

A) $f_3(x) = f(x_o) + f'(x_o)(x-x_o) + \frac{f''(x_o)(x-x_o)^2}{2!} + \frac{f'''(x_o)(x-x_o)^3}{3!}$

$= \frac{1}{x_o} - \frac{1}{x_o^2}(x-x_o) + \frac{1}{x_o^3}(x-x_o)^2 - \frac{1}{x_o^4}(x-x_o)^3$

For $x_o = 1/2$

$\underline{f_3(x) = 2 - 4(x-1/2) + 8(x-1/2)^2 - 16(x-1/2)^3}$

B) $f_3(0.6) = 2 - 4(0.1) + 8(0.1)^2 - 16(0.1)^3$

$= \underline{1.664}$

c) $f(0.6) = \frac{1}{0.6} = 1.66667$

$e_T(\%) = 100\left[\frac{f(0.6) - f_3(0.6)}{f(0.6)}\right] = \left[\frac{1.66667 - 1.664}{1.66667}\right] \times 100$

$\underline{e_T = 0.16\%}$

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EXAM 1 SOLUTIONS

SP 95

(2) $f(x) = x^3 - \frac{16}{x} = 0$

$f'(x) = 3x^2 + \frac{16}{x^2}$

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$, $i = 0, 1, 2, 3, \ldots$

i	x_i	$f(x_i)$	$f'(x_i)$	$\lvert e_A \rvert$, %
0	1	-15	19	–
1	1.7895	-3.2105	14.6033	44.12
2	2.0093	0.1497	16.0749	10.94
3	2.0000	0	16	0.46

(3)

A)

i	x_i	$f(x_i)$	Δ	Δ^2	Δ^3
0	0	1	$a-1$	$3-a$	$\frac{1+a}{2}$
1	1	a	$5-a$	$\frac{9+a}{2}$	
2	2	5	14		
3	3	19			

$$b_3 = \frac{1+a}{2} = 1 \quad , \quad a = 1$$

B)

$$f_3(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$= \underline{1 + 2x(x-1) + x(x-1)(x-2)}$$

$$= 1 + 2x^2 - 2x + x^3 - 3x^2 + 2x$$

$$= x^3 - x^2 + 1$$

(4)

x	y
1	1
2	3
3	6
4	12

$$f(x) = \begin{cases} 2x-1 & 1 \le x \le 2 \\ ax^2+bx+c & 2 \le x \le 3 \\ 2x^2-8x+12 & 3 \le x \le 4 \end{cases}$$

Equating slopes at interior pts

@ $x=2$, $\quad 2 = 2ax+b \big|_{x=2}$
$\quad = 4a+b$

@ $x=3$, $\quad 2ax+b \big|_{x=3} = 4x-8 \big|_{x=3}$

$6a+b = 4$

$$\begin{aligned} 4a+b &= 2 \\ 6a+b &= 4 \\ \hline 2a &= 2 \\ a &= 1 \\ b &= -2 \end{aligned}$$

At $x=2$, $\quad x^2-2x+c = 3$

$4-4+c = 3$

$c = 3$

<u>Check:</u>

At $x=3$, $\quad x^2-2x+3 = 6$

$9-6+3 = 6$ ✓

Ans. $a=1$
$b=-2$
$c=3$