Sp 95 Exam 1A Name _____

EGN 3420 **SHOW ALL WORK!**

Problem 1 (25 pts)

Consider the function $f(x) = x^{10} - 1$.

A) Find an expression for the 2nd order truncated Taylor Series Expansion $f_2(x)$ evaluated at $x=1.01x_0$, where x_0 is the point at which the series is expanded about. Leave your simplified answer in terms of x_0. Numeric constants should be rounded to 5 places after the decimal point.

B) If $x_0=1$, find the true relative error, as a per cent, in $f_2(1.01x_0)$.

C) If $x_0=1$, find $f_3(1.01x_0)$.

In Parts B) and C) express your answer rounded to 5 places after the decimal point.

.....WORK AREA.....

.....ANSWERS.....

A) $f_2(1.01x_0) =$ _____ , $e_T =$ _____ %, $f_3(1.01x_0) =$ _____

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Problem 2 (25 pts)

Use the simple one point iteration method to find the root of

$$f(x) = x - \ln(x^2 + 2) = 0$$

Start with $x_0 = 0$ and fill in the table below. Round all entries in the table to 4 places after the decimal point. Do not round the results of intermediate calculations.

i	x_i	$f(x_i)$
0	0	
1		
2		
5		
10		

.....WORK AREA.....

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Problem 3 (25 pts)

Complete the first four iterations of the Bisection Method to find the positive root of $f(x) = x^4 - 16$. Complete the table below. Round all answers to 4 places after the decimal point.

x_L	x_U	x_R	$\lvert e_T \rvert$, %
0	5		

.....WORK AREA.....

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Problem 4 (25 pts)

A quadratic spline fit thru the data points (1,1), (2,3), (3,6) and (4,12) is given below.

$$f(x) = \begin{cases} 2x - 1 & 1 \le x \le 2 \\ ax^2 + bx + c & 2 \le x \le 3 \\ 2x^2 - 8x + 12 & 3 \le x \le 4 \end{cases}$$

Find a, b, and c.

.....WORK AREA.....

.....ANSWERS.....

a = _____, b = _____, c = _____

Sp 95. EGN 4320 Exam 1 Solutions

① $f(x) = (x^5-1)(x^5+1)$

$= x^{10}-1$

$f'(x) = 10x^9$

$f''(x) = 90x^8$

$f'''(x) = 720x^7$

A) $f_2(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$

$= (x_0^{10}-1) + 10x_0^9(x-x_0) + 45x_0^8(x-x_0)^2$

$f_2(1.01x_0) = (x_0^{10}-1) + 10x_0^9(0.01x_0) + 45x_0^8(0.01x_0)^2$

$= x_0^{10} - 1 + 0.1x_0^{10} + 0.0045x_0^{10}$

$= \underline{1.1045x_0^{10} - 1}$

B) For $x_0 = 1$, $f_2(1.01) = 1.1045(1)^{10} - 1$

$= 0.10450$

$f(1.01) = (1.01)^{10} - 1$

$= 0.10462$

$e_T = \left[\frac{f(1.01) - f_2(1.01)}{f(1.01)}\right] \times 100$

$= \left[\frac{0.10462 - 0.10450}{0.10462}\right] \times 100$

$= \underline{0.11470\%}$

c) $f_3(1.01) = f_2(1.01) + \frac{f'''(1)}{3!}(0.01)^3$

$= 0.10450 + 120(0.000001)$

$= \underline{0.10462}$

Sp 95 EGN 3420 Exam 1 Solutions

(2) $f(x) = x - \ln(x^2+2)$

$g(x) = \ln(x^2+2)$

$x_{i+1} = g(x_i)$, $i = 0, 1, 2, 3, \ldots$

$x_1 = g(x_0) = \ln(x_0^2+2) = \ln(2) = 0.693147181$

$x_2 = g(x_1) = \ln(x_1^2+2) = \ln(2.480453014) = 0.90844121$

$x_3 = g(x_2) = \ln(x_2^2+2) = \ln(2.825265433) = 1.038602319$

$x_4 = g(x_3) = \ln(x_3^2+2) = \ln(3.078694776) = 1.124505733$

$x_5 = g(x_4) = \ln(x_4^2+2) = \ln(3.264513144) = 1.183110638$

$x_6 =$ 1.223702129

$x_7 =$ 1.252033246

$x_8 =$ 1.271889526

$x_9 =$ 1.285839285

$x_{10} =$ 1.295653496

i	x_i	$f(x_i)$
0	0	-0.6931
1	0.6931	-0.2153
2	0.9084	-0.1302
5	1.1831	-0.0406
10	1.2957	-0.0069

③ $f(x) = x^4 - 16$

$x_L = 0, x_U = 5$

x_L	x_U	x_R	$\|\epsilon_T\|, \%$
0	5	2.5	25
0	2.5	1.25	37.5
1.25	2.5	1.875	6.25
1.875	2.5	2.1875	9.375

④

i	x_i	$f(x_i)$	Δ	Δ^2	Δ^3
0	0	-1	1	0.5	0
1	2	1	2.5	0.5	
2	3	3.5	3.5		
3	4	7			

$b_0 = -1$

$b_1 = 1$

$b_2 = 0.5$

$b_3 = 0$

$f_3(x) = -1 + x + \frac{1}{2}x(x-2)$

$= \frac{x^2}{2} - 1$

$f_3(1) = \frac{1}{2} - 1$

$= -\frac{1}{2}$