Sp 94
EGN 3420 Exam 2 Name SOLNS

Problem 1 (50 pts)
Test scores of several students in two classes are given below.

Geometry (G)	Chemistry (C)
50	50
68	72
93	100
74	82
86	93
60	67

A) **Find the least squares line for predicting Chemistry scores (C) from Geometry scores (G).**

$6a_0 + 431a_1 = 464$
$431a_0 + 32{,}245a_1 = 34{,}782$
Soln. $a_0 = -3.8088, a_1 = 1.1296$

Ans. C = -3.8088 + 1.1296 G

B) **Find the coefficient of determination r^2 and the standard error of the estimate $s_{y|x}$.**

$$r^2 = \frac{SST - SSE}{SST} = \frac{1663.34 - 23.8914}{1663.34} = 0.9856$$

Ans. $r^2 = 0.9856$, $s_{y|x} = 2.444$

C) **Use the resulting least squares regression line to find the expected score in Chemistry for a student who made 80 on a Geometry test.**

Ans. $\hat{C} = 86.56$

$\hat{C} = -3.8088 + 1.1296(80) = 86.56$

Work Area

G_i	C_i	G_i^2	G_iC_i	$\hat{C}_i$	$(C_i-\hat{C}_i)^2$	$(C_i-\bar{C})^2$
50	50	2500	2500	52.67	7.1289	746.93
68	72	4624	4896	73.00	1.0000	28.41
93	100	8649	9300	101.24	1.5376	513.93
74	82	5476	6068	79.78	4.9284	21.81
86	93	7396	7998	93.34	0.1156	245.55
60	67	3600	4020	63.97	9.1809	106.71
431	464	32,245	34,782		23.8914 (SSE)	1663.34 (SST)

$\bar{C} = \frac{464}{6} = 77.33$

$$s_{Y|X} = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{23.8914}{4}} = 2.444$$

Problem 2 (25 pts)
The following data was obtained from an unknown function f(x).

i	x_i	$y_i = f(x_i)$
0	-1	1
1	0	1
2	1	1
3	2	5

A) Find the Lagrange interpolating polynomial

$f_3(x) = \sum f(x_i) L_i(x)$

You do not have to simplify the answer.

Ans. $f_3(x) =$

B) Estimate f(1.1).

Ans. $f(1.1) =$ 1.154

A) $$L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{x(x-1)(x-2)}{(-1-0)(-1-1)(-1-2)} = \frac{-1}{6}(x^3-3x^2+2x)$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x+1)(x-1)(x-2)}{(0+1)(0-1)(0-2)} = \frac{1}{2}(x^3-2x^2-x+2)$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x+1)(x)(x-2)}{(1+1)(1-0)(1-2)} = \frac{-1}{2}(x^3-x^2-2x)$$

$$L_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x+1)(x)(x-1)}{(2+1)(2-0)(2-1)} = \frac{1}{6}(x^3-x)$$

$$f_3(x) = Y_0L_0(x) + Y_1L_1(x) + Y_2L_2(x) + Y_3L_3(x)$$
$$= \frac{-1}{6}(x^3-3x^2+2x) + \frac{1}{2}(x^3-2x^2-x+2) - \frac{1}{2}(x^3-x^2-2x) + \frac{5}{6}(x^3-x)$$

B) $f_3(1.1) = 1.154$

Problem 3 (25 pts)
Given the data points (0,0), (1,0), (2,6) and (3,y_4), the resulting third order Newton Divided Difference polynomial is

$$f_3(x) = 3x(x-1) + x(x-1)(x-2)$$

Find y_4.

Ans. $y_4 =$ 24

$f_3(3) = 3(3)(3-1) + 3(3-1)(3-2) = y_4$

$18 + 6 = y_4$

$y_4 = 24$