EGN 3420 EXAM I Sp 94

Name Key

SHOW ALL WORK!

Problem 1

Consider the function $f(x) = x^5 - 1$.

A) Find an expression for the difference $f(x_1) - f_2(x_1)$ where $f_2(x)$ is the second order truncated Taylor Series expansion of $f(x)$ about some point x_0. Leave your answer in terms of x_0 and x_1.

B) Evaluate the expression from Part A) when $x_0=1$ and $x_1=1.1$

A) $f(x) = x^5 - 1$

$f'(x) = 5x^4$

$f''(x) = 20x^3$

$$f_2(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$$

$$= (x_0^5 - 1) + 5x_0^4(x-x_0) + \frac{20x_0^3}{2!}(x-x_0)^2$$

$$f(x_1) - f_2(x_1) = (x_1^5 - 1) - \{(x_0^5 - 1) + 5x_0^4(x_1-x_0) + 10x_0^3(x_1-x_0)^2\}$$

$$= (x_1^5 - x_0^5) - 5x_0^4(x_1-x_0) - 10x_0^3(x_1-x_0)^2 \quad \text{Ans.}$$

B) For $x_0 = 1$ & $x_1 = 1.1$

$$f(1.1) - f_2(1.1) = (1.1^5 - 1^5) - 5(1)^4(0.1) - 10(1)^3(0.1)^2$$

$$= 0.01051 \quad \text{Ans.}$$

Problem 2

The function $f(x) = x^4 - 10x^2 + 9$ has a root located between 2 and 5. Fill in the tables below for the first three iterations of the Bisection Method and the False Position method. Express all answers to four digits after the decimal point.

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	e_A
1	2	5	3.5	-15	36.5625	—
2	2	3.5	2.75	-15	-9.4336	-0.2727
3	2.75	3.5	3.125	-9.4336	6.7112	0.12

Bisection Method

Iteration 2, $e_A = \dfrac{x_r^{new} - x_r^{old}}{x_r^{new}} = \dfrac{2.75 - 3.5}{2.75} = -0.2727$

Iteration 3, $e_A = \dfrac{3.125 - 2.75}{3.125} = 0.12$

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_u)$	$f(x_r)$	e_A
1	2	4	2.25	-15	105	-15.9961	—
2	2.25	4	2.4814 ~~2.5475~~	-15.9961	105	-14.6606 ~~-13.7806~~	0.0933 ~~0.1168~~
3	~~2.5475~~ 2.4814	4	~~2.7160~~ 2.6675	~~-13.7806~~ -14.6606	105	~~-10.3515~~ -11.5244	~~0.0620~~ 0.0698

False Position Method

Iteration 1, $x_r = x_L - f(x_L)\left\{\dfrac{x_u - x_L}{f(x_u) - f(x_L)}\right\} = 2 - (-15)\left(\dfrac{4-2}{105 - -15}\right) = 2.25$

Iteration 2, $x_r = 2.25 - (-15.9961)\left(\dfrac{4 - 2.25}{105 - -15.9961}\right) = 2.\cancel{5475}\ 4814$

$e_A = \dfrac{x_r^{new} - x_r^{old}}{x_r^{new}} = \dfrac{2.\cancel{5475}\ 4814 - 2.25}{2.\cancel{5475}\ 4814} = \cancel{0.1168}\ 0.0933$

Iteration 3, $x_r = 2.\cancel{5475}\ 4814 - (-\cancel{13.7806}\ 14.6606)\left(\dfrac{4 - 2.\cancel{5475}\ 4814}{105 - - \cancel{13.7806}\ 14.6606}\right) = 2.7160$

$e_A = \dfrac{\cancel{2.7160 - 2.5475}\ 2.6675 - 2.4814}{\cancel{2.7160}\ 2.6675} = \cancel{0.0620}\ 0.0698$

Problem 3

Consider the function $f(x) = x^3 + x - 1$.

A) Use the Simple One Point Iteration Method and fill in the table below. Be certain that the g(x) function you choose satisfies the condition $|g'(x)| < \theta$ for all x.

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i	x_i	$x_{i+1} = g(x_i)$
0	0	1
1	1	0.5
2	0.5	0.8

$f(x) = x(x^2+1) - 1 = 0$

$\Rightarrow x = g(x) = \frac{1}{x^2+1}$

$g'(x) = \frac{-2x}{(x^2+1)^2}$

$|g'(x)| = \frac{2|x|}{(x^2+1)^2} < 1$ for $-\infty < x < \infty$

$x_{i+1} = g(x_i) = \frac{1}{x_i^2+1}$

$x_1 = \frac{1}{x_0^2+1} = \frac{1}{0+1} = 1$

$x_2 = \frac{1}{x_1^2+1} = \frac{1}{1^2+1} = 0.5$

$x_3 = \frac{1}{x_2^2+1} = \frac{1}{(0.5)^2+1} = 0.8$

B) Use the Newton-Raphson Method and fill in the table below.

i	x_i	$f(x_i)$	$f'(x_i)$	x_{i+1}
0	0	-1	1	1
1	1	1	4	0.75

$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$, $f'(x) = 3x^2 + 1$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{(-1)}{1}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{1}{4} = 0.75$

Problem 4

Evaluate the following determinant:

$$\begin{vmatrix} 1 & 0 & 1 & 3 \\ -2 & 4 & 0 & 1 \\ 7 & 6 & 1 & 2 \\ 7 & 2 & -1 & 5 \end{vmatrix}$$

Be sure to show the results of intermediate steps.

$$\begin{vmatrix} 1 & 0 & 1 & 3 \\ -2 & 4 & 0 & 1 \\ 7 & 6 & 1 & 2 \\ 7 & 2 & -1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 1 & 3 \\ 0 & 4 & 2 & 7 \\ 0 & 6 & -6 & -19 \\ 0 & 2 & -8 & -16 \end{vmatrix} = \begin{vmatrix} 4 & 2 & 7 \\ 6 & -6 & -19 \\ 2 & -8 & -16 \end{vmatrix}$$

$$= -\begin{vmatrix} 2 & -8 & -16 \\ 6 & -6 & -19 \\ 4 & 2 & 7 \end{vmatrix} = -(2)(\cancel{8}) \begin{vmatrix} 1 & -4 & -8 \\ \cancel{86} & -\cancel{86} & -\frac{19}{3} \\ 4 & 2 & 7 \end{vmatrix} = -\frac{2}{\cancel{6}} \begin{vmatrix} 1 & -4 & -8 \\ 0 & 18 & \frac{29}{\cancel{13}} \\ 0 & 18 & 39 \end{vmatrix}$$

$$= -\frac{2}{\cancel{6}} \begin{vmatrix} \frac{18}{\cancel{6}} & \frac{29}{\cancel{13}} \\ 18 & 39 \end{vmatrix} = -\frac{2}{\cancel{6}}\left(\overset{702}{\cancel{3 \cdot 8 \cdot 4}} - \overset{522}{\cancel{12 \cdot 3 \cdot 4}}\right) = \overset{-360}{\cancel{}}$$

Ans.

Problem 5

Solve the system of equations below by using the inverse matrix.

$$\begin{aligned} x + \phantom{2y - {}} z &= 1 \\ 2y - z &= 1 \\ 3x + y + 4z &= 1 \end{aligned}$$

$A\underline{x} = \underline{b}$ where $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 3 & 1 & 4 \end{pmatrix}$, $\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\underline{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$\underline{x} = A^{-1}\underline{b}$

$A^{-1} = \frac{1}{|A|} \operatorname{Adj} A$ $\qquad |A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 3 & 1 & 4 \end{vmatrix} = 1(9) + 3(-2) = 3$

$$A^{c} = \begin{pmatrix} 9 & -3 & -6 \\ 1 & 1 & -1 \\ -2 & 1 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{pmatrix} 9 & 1 & -2 \\ -3 & 1 & 1 \\ -6 & -1 & 2 \end{pmatrix}$$

$$\begin{aligned} \underline{x} &= A^{-1}\underline{b} \\ &= \frac{1}{3}\begin{pmatrix} 9 & 1 & -2 \\ -3 & 1 & 1 \\ -6 & -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \\ &= \frac{1}{3}\begin{pmatrix} 8 \\ -1 \\ -5 \end{pmatrix} \end{aligned}$$

$\Rightarrow$ $x = 8/3$, $y = -1/3$, $z = -5/3$ Ans.