For the table of data points given below

x_i	y_i	$u_i = \log x_i$	$z_i = \log y_i$	u_i^2	$u_i z_i$
1	1	0	0	0	0
2	6	0.3010	0.7782	0.0906	0.2342
3	13	0.4771	1.1139	0.2276	0.5315
4	24	0.6021	1.3802	0.3625	0.8310
5	40	0.6990	1.6021	0.4886	1.1198
		$\sum u_i = 2.0792$	$\sum z_i = 4.8744$	$\sum u_i^2 = 1.1693$	$\sum u_i z_i = 2.7165$

i) A power equation model $y = ax^b$ is used for interpolation. Fill in the table and solve for the coefficients a and b.   (8 pts)

a) $a = 0.8615,\quad b = 2.5405$ b) $a = 1.2582,\quad b = 2.3559$ c) $a = 1.0810,\quad b = 2.2631$
d) $a = 1.5000,\quad b = 2.0000$ e) $a = 0.7505,\quad b = 2.7519$ d) none of the above

$z = a_0 + a_1 u$

$$na_0 \quad + \quad \left(\sum u_i\right)a_1 \quad = \quad \sum z_i$$

$$\left(\sum u_i\right)a_0 \quad + \quad \left(\sum u_i^2\right)a_1 \quad = \quad \left(\sum u_i z_i\right)$$

$$5a_0 \quad + \quad 2.0792a_1 \quad = \quad 4.8744$$

$$2.0709a_0 \quad + \quad 1.1693a_1 \quad = \quad 2.7165$$

$a_0 = 0.0338, \quad a_1 = 2.2631$

$a = 10^{a_0} = 10^{0.0338} = 1.0810$

$b = a_1 = 2.2631$

ii) Use the power equation to estimate y when $x = 4$. The result is   (2 pts)

a) 23.15   b) 24.91   c) 22.84   d) 24   e) 24.54   f) 23.75   g) none of the above

$y_{est} = ax^b = 1.0810(4^{2.2631}) = 24.9063$

x =   1   2   3   4   5

y =   1   6   13   24   40

u =   0   0.3010   0.4771   0.6021   0.6990

z =   0   0.7782   1.1139   1.3802   1.6021

usq =   0   0.0906   0.2276   0.3625   0.4886

uz =   0   0.2342   0.5315   0.8310   1.1198

sum_u =   2.0792

sum_z =   4.8744

sum_usq =   1.1693

sum_uz =   2.7165

n =   5

A =

  5.0000   2.0792

  2.0792   1.1693

b =

  4.8744

  2.7165

a =

  0.0338

  2.2631

a0 =   0.0338

a1 =   2.2631

a =   1.0810

b =   2.2631

y_est =   24.9063