Problem 1

Given the system of equations \( Ax = b \),

\[
\begin{align*}
    x + y + z &= 0 \\
    2x + z &= 1 \\
    2y + z &= -1 \\
    x - y - z &= 2
\end{align*}
\]

Find the Echelon form of \( (A|b) \) and choose the correct answer from the choices below.

a) The system of equations are inconsistent
b) The system of equations are consistent with a unique solution \( x=1, \ y=1, \ z=1 \)
c) The system of equations are consistent with an infinite number of solutions
d) The system of equations are consistent with a unique solution \( x=1, \ y=0, \ z=-1 \)

\[
(A|b) = \begin{bmatrix}
1 & 1 & 1 & 0 \\
2 & 0 & 1 & 1 \\
0 & 2 & 1 & -1 \\
1 & -1 & -1 & 2
\end{bmatrix} \sim \begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & -2 & -1 & 1 \\
0 & 2 & 1 & -1 \\
0 & -2 & -2 & 2
\end{bmatrix}
\]

Divide last row by -2 and swap with 2nd row

\[
\sim \begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & 1 & 1 & -1 \\
0 & 2 & 1 & -1 \\
0 & -2 & -1 & 1
\end{bmatrix} \sim \begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & 1 & 1 & -1 \\
0 & 0 & -1 & 1 \\
0 & 0 & 1 & -1
\end{bmatrix} \sim \begin{bmatrix}
1 & 1 & 1 & 0 \\
0 & 1 & 1 & -1 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

The equations are consistent with a unique solution. The solution is found from

\[
\begin{align*}
    z &= -1 \\
    y + z &= -1 \quad \Rightarrow \quad y - 1 = -1 \quad \Rightarrow \quad y = 0 \\
    x + y + z &= 0 \quad \Rightarrow \quad x + 0 - 1 = 0 \quad \Rightarrow \quad x = 1
\end{align*}
\]
Consider the Matlab script file

```
A=[1 2; 3 4]
for i=1:3
    A=2*A
    if A(1,1)==8
        x=1
    else
        x=0
    end
end
```

Running the script file results in

a) an error message
b) x=0
c) x=1
d) x=2
e) x=3
f) none of the above

```
A =
    1  2
    3  4
A =
    2  4
    6  8
x =  0
A =
    4  8
    12 16
x =  0
A =
    8  16
    24 32
x =   1
```