

1. Given the following matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (3 pts)

its inverse A^{-1} is

a) $\begin{bmatrix} 1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$ b) $\begin{bmatrix} -1/3 & -2/3 \\ 2/3 & -1/3 \end{bmatrix}$ c) $\begin{bmatrix} -1/3 & 2/3 \\ 2/3 & 1/3 \end{bmatrix}$ d) $\begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$

e) none of the above

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad |A| = \begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = 1 - 4 = -3$$

$$\text{cof}(a_{11}) = (-1)^{1+1}(1) = 1$$

$$\text{cof}(a_{12}) = (-1)^{1+2}(2) = -2$$

$$\text{cof}(a_{21}) = (-1)^{2+1}(2) = -2$$

$$\text{cof}(a_{22}) = (-1)^{2+2}(1) = 1$$

$$A^c = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}, \quad \text{Adj } A = (A^c)^T = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{-3} \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$$

2. The system of equations (2 pts)

$$-2x + 3y + z = 25$$

$$-5x + 4y + 2z = -75$$

$$3x - z = 100$$

- a) is inconsistent
 b) is consistent with a unique solution
 c) is consistent with infinite solutions

$$\underline{Ax} = \underline{b}, \quad A = \begin{bmatrix} -2 & 3 & 1 \\ -5 & 4 & 2 \\ 3 & 0 & -1 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 25 \\ -75 \\ 100 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -2 & 3 & 1 \\ -5 & 4 & 2 \\ 3 & 0 & -1 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & -2 \\ 2 & 4 & -5 \\ -1 & 0 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & -2 \\ 0 & -2 & -1 \\ 0 & 3 & 1 \end{vmatrix} = - \begin{vmatrix} -2 & -1 \\ 3 & 1 \end{vmatrix} = -(-2+3) = -1$$

Since $|A| \neq 0$, the system of equations $\underline{Ax} = \underline{b}$ is consistent with a unique solution.

3. Consider the Matlab script file

(5 pts)

```
A=[1;2]
B=[2 1]
det_AB=det(A*B)
v=0:3
v=v/det_AB
```

Running the script file results in

- a) An error message
- b) $v = 0 \ 0 \ 0 \ 0$
- c) $v = 0 \ 1 \ 2 \ 3$
- d) $v = \text{NaN} \ 0 \ 0 \ 0$
- e) $v = \text{NaN} \ \text{inf} \ \text{inf} \ \text{inf}$
- f) $v = \text{inf} \ \text{inf} \ \text{inf} \ \text{inf}$
- g) none of the above

```
A =
     1
     2
```

```
B =     2     1
```

```
det_AB =     0
```

```
v =     0     1     2     3
```

```
v = NaN -Inf -Inf -Inf
```