Problem 1

Given the function \( f(x) = e^{2x} \), find the second order truncated Taylor Series \( f_2(x) \) expanded about the point \( x_0 = 0 \). Use \( f_2(0.5) \) to approximate the true value \( f(0.5) \). The numerical value of \( f_2(0.5) \) is

- a) 0
- b) 1
- c) 2.5
- d) 2.7183
- e) 3
- f) none of the above (3 pts)

\[
f(x) = e^{2x}, \quad f(x_0) = e^{2x_0} = e^{2(0)} = 1
\]

\[
f'(x) = 2e^{2x}, \quad f'(x_0) = 2e^{2x_0} = 2e^{2(0)} = 2
\]

\[
f''(x) = 4e^{2x}, \quad f''(x_0) = 4e^{2x_0} = 4e^{2(0)} = 4
\]

\[
f_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2
\]

\[
= 1 + 2(x - 0) + \frac{4}{2}(x - 0)^2
\]

\[
= 1 + 2x + 2x^2
\]

\[
f_2(0.5) = 1 + 2(0.5) + 2(0.5)^2
\]

\[
= 2.5
\]

The true error \((E_T)\) of the estimate \( f_2(0.5) \) is

- a) 2.7183
- b) 1.7183
- c) 0
- d) 0.2183
- e) -0.2817
- f) none of the above (2 pts)

\[
E_T = f(0.5) - f_2(0.5)
\]

\[
= e^{2(0.5)} - 2.5
\]

\[
= 2.7183 - 2.5
\]

\[
= 0.2183
\]
Problem 2

Consider the following Matlab script file:

```matlab
x=0:2:10
y=1:6
w=x.*y
z=x+y+w
a=sum(z)/6
```

After running the script file, the numerical value of a is

a) No value because an error message will appear   b) 21.3281   c) 14.7103

d) 35.2952   e) 31.8333   f) none of the above

```
x =  0  2  4  6  8  10
y =  1  2  3  4  5  6
w =  0  4  12  24  40  60
z =  1  8  19  34  53  76
a = 31.8333
```