EGN 3420 Exam 2, Part 1 solution Sp 2011

Do Any 7 problems! (No calculators)
Write your answers on the Answer Sheet only after you are sure of your answer.
Make sure you have only 7 answers on the Answer Sheet!
All problems are worth 10 pts

1. The system of equations $\begin{matrix} x+y=0 \\ x-y=1 \end{matrix}$ is

a) consistent with a single solution b) consistent with infinite solutions c) inconsistent

$$A\underline{x}=\underline{b}, \quad A=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}, \quad \underline{b}=\begin{bmatrix}0\\ 1\end{bmatrix}, \quad \underline{x}=\begin{bmatrix}x\\ y\end{bmatrix}$$

$$|A|=\begin{vmatrix}1 & 1\\ 1 & -1\end{vmatrix}=-1-1=-2$$

Since $|A|$ is not zero, the equations are consistent with a unique solution.

2. The system of equations $\begin{matrix} x & + & y & = & 1 \\ x & + & 2y & = & 0 \\ 2x & - & y & = & 1 \end{matrix}$ is

a) consistent with a single solution b) consistent with infinite solutions c) inconsistent

$$A\underline{x}=\underline{b}, \quad A=\begin{bmatrix}1 & 1\\ 1 & 2\\ 2 & -1\end{bmatrix}, \quad \underline{b}=\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$

$$(A|\underline{b})=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 0\\ 2 & -1 & 1\end{bmatrix} \sim \begin{bmatrix}1 & 1 & 1\\ 0 & 1 & -1\\ 0 & -3 & -1\end{bmatrix} \sim \begin{bmatrix}1 & 1 & 1\\ 0 & 1 & -1\\ 0 & 0 & -4\end{bmatrix}$$

From the last row of the echelon form, the equations are inconsistent.

3. The system of equations $\begin{matrix} x & + & y & + & z & = & 2 \\ x & + & 2y & - & z & = & 0 \\ x & - & y & + & 2z & = & K \end{matrix}$ is

a) inconsistent for all values of *K*
b) consistent for all values of *K*
c) consistent with solution $x=1,\ y=0,\ z=1$ when $K=1$
d) none of the above

$$A\underline{x}=\underline{b},\qquad A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix},\qquad \underline{b}=\begin{bmatrix}2\\ 0\\ K\end{bmatrix},\qquad \underline{x}=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$(A\,|\,\underline{b})=\begin{bmatrix}1 & 1 & 1 & 2\\ 1 & 2 & -1 & 0\\ 1 & -1 & 2 & K\end{bmatrix}\sim\begin{bmatrix}1 & 1 & 1 & 2\\ 0 & 1 & -2 & -2\\ 0 & -2 & 1 & K-2\end{bmatrix}\sim\begin{matrix}x & y & z & \\ \end{matrix}\begin{bmatrix}1 & 1 & 1 & 2\\ 0 & 1 & -2 & -2\\ 0 & 0 & -3 & K-6\end{bmatrix}$$

Since the first 3 columns of $(A\,|\,\underline{b})$ are in upper triangular form, $|A|\neq 0$ and the equations are consistent for all values of *K*. From the last equation,

$$-3z=K-6\qquad\Rightarrow\qquad z=\frac{6-K}{3}=\frac{5}{3}\qquad \text{when } K=1$$

4. The same system of equations as in Problem 3,
$$\begin{aligned}x &+ y &+ z &= 2\\ x &+ 2y &- z &= 0\\ x &- y &+ 2z &= K\end{aligned}$$
is

a) inconsistent for all values of *K*
b) consistent with solution $x=1,\ y=0,\ z=1$ when $K=0$
c) consistent with solution $x=1,\ y=0,\ z=1$ when $K=3$
d) consistent with solution $x=1,\ y=0,\ z=1$ when $-\infty<K<\infty$
e) none of the above

$$\text{For } K=0,\qquad z=\frac{6-K}{3}=\frac{6}{3}=2$$

$$\text{For } K=3,\qquad z=\frac{6-K}{3}=\frac{6-3}{3}=1$$

$$y-2z=-2\qquad\Rightarrow\qquad y=2z-2=2(1)-2=0$$

$$x+y+z=2\qquad\Rightarrow\qquad x=2-y-z=2-0-1=1$$

$$x=1,y=0,z=1$$

5. The system of equations
$$\begin{array}{ccccccc} a & + & b & + & 2c & = & 3 \\ a & & & + & c & = & 2 \\ & & b & + & c & = & 1 \end{array}$$

a) has a unique solution
b) is inconsistent
c) is consistent with one arbitrary unknown
d) none of the above

$$A\underline{x} = \underline{b}, \quad A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$(A|\underline{b}) = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \sim \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The equations are consistent with one arbitrary unknown.

6. Given matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$,

a) A^{-1} does not exist b) $A^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ c) $A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$ d) $A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & -1 \end{bmatrix}$

e) $A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ f) none of the above

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad |A| = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1$$

$$cof\,A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}, \quad Adj\,A = (cof\,A)^T = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} Adj\,A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

7. After two iterations using the Jacobi Method to solve $\begin{aligned} 2x + y &= 1 \\ x + 2y &= -1 \end{aligned}$ with initial guess $x_0 = 0,\ y_0 = 0$ results in

a) $x_2 = 0.75,\ y_2 = -0.75$ b) $x_2 = 0.75,\ y_2 = -0.5$

c) $x_2 = -0.75,\ y_2 = 0.75$ d) $x_2 = 0.5,\ y_2 = -0.75$ e) none of the above

$$2x + y = 1 \Rightarrow x = \frac{1-y}{2}, \qquad x_0 = 0$$

$$x + 2y = -1 \Rightarrow y = \frac{-1-x}{2}, \qquad y_0 = 0$$

$$x_1 = \frac{1-y_0}{2} = \frac{1-0}{2} = \frac{1}{2}, \qquad y_1 = \frac{-1-x_0}{2} = \frac{-1-0}{2} = -\frac{1}{2}$$

$$x_2 = \frac{1-y_1}{2} = \frac{1-(-0.5)}{2} = 0.75, \qquad y_2 = \frac{-1-x_1}{2} = \frac{-1-(0.5)}{2} = -0.75$$

8. Given the same system of equations in Problem 7 $\begin{aligned} 2x + y &= 1 \\ x + 2y &= -1 \end{aligned}$ the solution for x is $x = 1.$ Using the Jacobi Method with initial guess $x_0 = 0,\ y_0 = 0$ results in a true error in x, $\left(E_T\right)_x$ after the second iteration of

a) - 0.5 b) 0 c) 0.75 d) 0.25 e) 1 f) none of the above

$$\left(E_T\right)_x = 1 - x_2 = 1 - 0.75 = 0.25$$