EGN 3420 Exam I solution Sp 2011

Do Any 7 of the first 9 Problems
You must do Problems 10,11,12
Write your answers on the Answer Sheet.
All problems are worth 10 pts

Note: On some problems, your answer might be slightly different than the correct answer due to round off of calculations using a calculator. If you do not find the correct answer from the choices a) – e), enter f) on the Answer Sheet.

Given the function, $f(x) = xe^{-x}$ (Problems 1 – 4)

1. The second term (not the second order term) in the Taylor Series Expansion of $f(x)$ about $x_0 = 0$ is

   a) $x - 1$ b) x c) $e^{-x}(1 - x)$ d) xe^{-x} e) 0

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots.. \quad \text{Taylor Series expansion of } f(x)$$

$$f'(x) = x(-e^{-x}) + (1)(e^{-x}) = e^{-x}(1 - x)$$

$$f'(x_0)(x - x_0) = e^{-x_0}(1 - x_0)(x - x_0), \quad x_0 = 0$$

$$= e^{-0}(1 - 0)(x - 0)$$

$$= x$$

2. The final term in $f_2(x)$, the second order truncated Taylor Series Expansion of $f(x)$ about the point $x_0 = 0$ is

   a) $e^{-x}(x - 2)$ b) $-x^2/2$ c) x^2 d) $-x^2$ e) $x^2/2$

$$f''(x) = e^{-x}(-1) + (-e^{-x})(1 - x)$$

$$= e^{-x}(x - 2)$$

$$\frac{f''(x_0)}{2!}(x - x_0)^2 = \frac{e^{-x_0}(x_0 - 2)}{2}(x - x_0)^2, \quad x_0 = 0$$

$$= \frac{e^{-0}(0 - 2)}{2}(x - 0)^2$$

$$= -x^2$$

3. The first order truncated Taylor Series Expansion of $f(x)$ about the point $x_0 = 0$ is denoted by $f_1(x)$. The error $E_T = f(0.1) - f_1(0.1)$ in $f_1(0.1)$ is

a) 0.2202 b) -0.0153 c) -0.0095 d) -0.0095 e) 0

$$f_1(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$= x_0 e^{-x_0} + e^{-x_0}(1 - x_0)(x - x_0), \quad x_0 = 0$$

$$= x$$

$$f_1(0.1) = 0.1$$

$$f(0.1) = 0.1e^{-0.1} = 0.0905$$

$$E_T = f(0.1) - f_1(0.1)$$

$$= 0.0905 - 0.1$$

$$= -0.0095$$

4. The approximate relative error $e_A = [f_2(0.1) - f_1(0.1)]/f_2(0.1)$ is

a -0.1111 b) 0.56 c) 0.0255 d) 0 e) -0.1847

$$f_2(x) = f_1(x) + \frac{f''(x_0)}{2!}(x - x_0)^2$$

$$= x - x^2 \quad \text{(from Parts 2 and 3)}$$

$$f_2(0.1) = 0.1 - (0.1)^2 = 0.1 - 0.01 = 0.09$$

$$e_A = \frac{f_2(0.1) - f_1(0.1)}{f_2(0.1)} = \frac{0.09 - 0.1}{0.09} = -0.1111$$

5. The second order truncated Taylor Series $f_2(x)$ of the function $f(x) = x^2 - 2x + 1$ expanded about $x_0 = 0$ is

a) $1 + x + x^2$ b) $1 - x^2$ c) $1 - 2x + x^2$ d) $1 - 2x + 2x^2$ e) 0

$$f(x) = x^2 - 2x + 1, \qquad f(x_0) = f(0) = 1$$

$$f'(x) = 2x - 2, \qquad f'(x_0) = f'(0) = -2$$

$$f''(x) = 2, \qquad f''(x_0) = f''(0) = 2$$

$$f_2(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2, \quad x_0 = 0$$

$$= 1 - 2(x - 0) + \frac{2}{2}(x - 0)^2$$

$$= 1 - 2x + x^2$$

6. For the same function $f(x)$ and $f_2(x)$ from Problem 5, the truncation error $E_T = f(2) - f_2(2)$ is

a) 1 b) -4 c) 0.1 d) -0.25 e) 0

$$f(2) = 2^2 - 2(2) + 1 = 1$$

$$f_2(2) = 1 - 2(2) + 2^2 = 1$$

$$E_T = f(2) - f_2(2) = 1 - 1 = 0$$

Consider the function $f(x) = e^{-x} - 1$. The Bisection method is used to find the real root of $f(x)$. (Problems 7 and 9)

7. For which of the following initial brackets will the method fail to converge?

a) $x_l = -1, x_u = 1$ b) $x_l = -10, x_u = 5$ c) $x_l = -0.1, x_u = 0.25$

d) $x_l = -100, x_u = 25$ e) $x_l = 0.1, x_u = 1$

$$f(x) = e^{-x} - 1 = 0 \quad \Rightarrow \quad e^{-x} = 1 \quad \Rightarrow \quad \ln(e^{-x}) = \ln(1) \quad \Rightarrow \quad -x = 0 \quad \Rightarrow \quad x = 0$$

The root of $f(x)$ is at $x = 0$. The last bracket $x_l = 0.1, x_u = 1$ does not contain the root and the Bisection method will fail to converge.

8. The initial bracket is $x_l = -1, x_u = 2$. After two iterations, x_R is

a) 0 b) -0.25 c) 0.5 d) 0.1 e) -0.5

$$x_R = \frac{x_l + x_u}{2} = \frac{-1 + 2}{2} = 0.5$$

$$f(x_l) = f(-1) = e^{-(-1)} - 1 = e - 1 = 1.7183$$

$$f(x_R) = f(0.5) = e^{-0.5} - 1 = -0.3935$$

$$f(x_l)f(x_R) = (1.7183)(-0.3935) < 0 \quad \Rightarrow \quad x_l = -1, \quad x_u = x_R = 0.5$$

$$x_R = \frac{x_l + x_u}{2} = \frac{-1 + 0.5}{2} = -0.25$$

9. After the second iteration, the true error $E_T = R - f(x_R)$ is

a) 0.1827 b) -0.0296 c) -0.2840 d) 0 e) 1.25

$$E_T = R - f(x_R) = 0 - f(-0.25) = -\left[e^{-(-0.25)} - 1\right] = -0.2840$$

10. Consider the following Matlab script file.

```
t=1:3;   x=2*t;   y=t*t;   z=x+y;
```

Running the script file will result in

a) z=[3 8 15] b) z=[1 0 -3] c) z=[15 15 15] d) an error message
e) z=[0 0 0]

Since t is the 1x3 array [1 2 3], the statement y=t*t will result in an error message because the operation of multiplying a 1x3 array times a 1x3 array is not valid.

11. Executing the Matlab statement x=linspace(0,5,6) - [0:5] in the Command Window will result in

a) an error message b) x = 0 0 0 0 0 0 c) x = 0 1 0 1 0 1
d) ans = 0 0 0 0 0 0 e) ans = 1 2 3 4 5

linspace(0,5,6) = [0 1 2 3 4 5] and [0:5] =[0 1 2 3 4 5]
x=linspace(0,5,6) - [0:5] = [0 1 2 3 4 5] – [0 1 2 3 4 5] =[0 0 0 0 0 0]

12. Executing the Matlab statements theta=[0 pi/2], x=cos(theta) – sin(theta), will result in

a) an error message b) x=[1.0000 -1.0000] c) x=[0.5000 1.0000]
d) x=[-1.0000 0] e) x=[0 0]

x =cos(theta) – sin(theta)
=cos([0 pi/2]) - sin([0 pi/2])
=[cos(0) cos(pi/2)] - [sin(0) sin(pi/2)]
=[1 0] - [0 1]
=[1 -1]