Do Any 7 of the first 9 Problems
You must do Problems 10, 11, 12
Write your answers on the Answer Sheet.
All problems are worth 10 pts

Note: On some problems, your answer might be slightly different than the correct answer due to round off of calculations using a calculator. If you do not find the correct answer from the choices a) – e), enter f) on the Answer Sheet.

Given the function, \( f(x) = xe^{-x} \) (Problems 1 – 4)

1. The second term (not the second order term) in the Taylor Series Expansion of \( f(x) \) about \( x_0 = 0 \) is
   a) \( x - 1 \)    b) \( x \)    c) \( e^{-x}(1 - x) \)    d) \( xe^{-x} \)    e) 0

   \[
   f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots \quad \text{Taylor Series expansion of } f(x)
   \]

   \[
   f'(x) = x(-e^{-x}) + (1)(e^{-x}) = e^{-x}(1 - x)
   \]

   \[
   f''(x_0)(x - x_0) = e^{-x_0}(1 - x_0)(x - x_0), \quad x_0 = 0
   \]

   \[
   = e^0(1 - 0)(x - 0)
   \]

   \[
   = x
   \]

2. The final term in \( f_2(x) \), the second order truncated Taylor Series Expansion of \( f(x) \) about the point \( x_0 = 0 \) is
   a) \( e^{-x}(x - 2) \)    b) \( -x^2/2 \)    c) \( x^2 \)    d) \( -x^2 \)    e) \( x^2/2 \)

   \[
   f''(x) = e^{-x}(-1) + (-e^{-x})(1 - x)
   \]

   \[
   = e^{-x}(x - 2)
   \]

   \[
   \frac{f''(x_0)}{2!}(x - x_0)^2 = \frac{e^{-x_0}(x_0 - 2)}{2}(x - x_0)^2, \quad x_0 = 0
   \]

   \[
   = \frac{e^0(0 - 2)}{2}(x - 0)^2
   \]

   \[
   = -x^2
   \]
3. The first order truncated Taylor Series Expansion of $f(x)$ about the point $x_0 = 0$ is denoted by $f_1(x)$. The error $E_T = f(0.1) - f_1(0.1)$ in $f_1(0.1)$ is

$$f_1(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$= x_0 e^{-x_0} + e^{-x_0} (1 - x_0)(x - x_0), \quad x_0 = 0$$

$$= x$$

$$f_1(0.1) = 0.1$$

$$f(0.1) = 0.1e^{-0.1} = 0.0905$$

$$E_T = f(0.1) - f_1(0.1)$$

$$= 0.0905 - 0.1$$

$$= -0.0095$$

a) 0.2202  b) -0.0153  c) -0.0095  d) -0.0095  e) 0

4. The approximate relative error $e_A = [f_2(0.1) - f_1(0.1)]/f_2(0.1)$ is

$$f_2(x) = f_1(x) + \frac{f''(x_0)}{2!}(x - x_0)^2$$

$$= x - x^2 \quad \text{(from Parts 2 and 3)}$$

$$f_2(0.1) = 0.1 - (0.1)^2 = 0.1 - 0.01 = 0.09$$

$$e_A = \frac{f_2(0.1) - f_1(0.1)}{f_2(0.1)} = \frac{0.09 - 0.1}{0.09} = -0.1111$$

a) -0.1111  b) 0.56  c) 0.0255  d) 0  e) -0.1111

5. The second order truncated Taylor Series $f_2(x)$ of the function $f(x) = x^2 - 2x + 1$ expanded about $x_0 = 0$ is

a) $1 + x + x^2$  b) $1 - x^2$  c) $1 - 2x + x^2$  d) $1 - 2x + 2x^2$  e) 0

$$f(x) = x^2 - 2x + 1, \quad f(x_0) = f(0) = 1$$

$$f'(x) = 2x - 2, \quad f'(x_0) = f'(0) = -2$$

$$f''(x) = 2, \quad f''(x_0) = f''(0) = 2$$
\[ f_2(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2, \quad x_0 = 0 \]
\[ = 1 - 2(x-0) + \frac{2}{2}(x-0)^2 \]
\[ = 1 - 2x + x^2 \]

6. For the same function \( f(x) \) and \( f_2(x) \) from Problem 5, the truncation error \( E_T = f(2) - f_2(2) \) is
a) 1  b) -4  c) 0.1  d) -0.25  e) 0

\[
\begin{align*}
f(2) &= 2^2 - 2(2)+1 = 1 \\
f_2(2) &= 1 - 2(2) + 2^2 = 1 \\
E_T &= f(2) - f_2(2) = 1 - 1 = 0
\end{align*}
\]

Consider the function \( f(x) = e^{-x} - 1 \). The Bisection method is used to find the real root of \( f(x) \). (Problems 7 and 9)

7. For which of the following initial brackets will the method fail to converge?
   a) \( x_l = -1, x_u = 1 \)  
   b) \( x_l = -10, x_u = 5 \)  
   c) \( x_l = -0.1, x_u = 0.25 \)  
   d) \( x_l = -100, x_u = 25 \)  
   e) \( x_l = 0.1, x_u = 1 \)

\[
\begin{align*}
f(x) &= e^{-x} - 1 = 0 \Rightarrow e^{-x} = 1 \Rightarrow \ln(e^{-x}) = \ln(1) \Rightarrow -x = 0 \Rightarrow x = 0
\end{align*}
\]

The root of \( f(x) \) is at \( x = 0 \). The last bracket \( x_l = 0.1, x_u = 1 \) does not contain the root and the Bisection method will fail to converge.

8. The initial bracket is \( x_l = -1, x_u = 2 \). After two iterations, \( x_R \) is
   a) 0  
   b) -0.25  
   c) 0.5  
   d) 0.1  
   e) -0.5

\[
\begin{align*}
x_R &= \frac{x_l + x_u}{2} = \frac{-1 + 2}{2} = 0.5 \\
f(x_l) &= f(-1) = e^{-(-1)} - 1 = e - 1 = 1.7183 \\
f(x_u) &= f(0.5) = e^{0.5} - 1 = -0.3935 \\
f(x_l)f(x_u) &= (1.7183)(-0.3935) < 0 \Rightarrow x_l = -1, x_u = x_R = 0.5 \\
x_R &= \frac{x_l + x_u}{2} = \frac{-1 + 0.5}{2} = -0.25
\end{align*}
\]
9. After the second iteration, the true error \( E_r = R - f(x_r) \) is
   
   a) 0.1827   b) -0.0296   c) -0.2840   d) 0   e) 1.25

   \[ E_r = R - f(x_r) = 0 - f(-0.25) = -[e^{-(-0.25)} - 1] = -0.2840 \]

10. Consider the following Matlab script file.

\[
t=1:3; \quad x=2*t; \quad y=t*t; \quad z=x+y;
\]

Running the script file will result in
   
   a) \( z=[3 \ 8 \ 15] \)   b) \( z=[1 \ 0 \ -3] \)   c) \( z=[15 \ 15 \ 15] \)   d) an error message
   e) \( z=[0 \ 0 \ 0] \)

Since \( t \) is the 1x3 array \([1 \ 2 \ 3]\), the statement \( y=t^t \) will result in an error message because the operation of multiplying a 1x3 array times a 1x3 array is not valid.

11. Executing the Matlab statement \( x=linspace(0,5,6) - [0:5] \) in the Command Window will result in
   
   a) an error message   b) \( x = 0 \ 0 \ 0 \ 0 \ 0 \ 0 \)   c) \( x = 0 \ 1 \ 0 \ 1 \ 0 \ 1 \)
   d) \( \text{ans} = 0 \ 0 \ 0 \ 0 \ 0 \ 0 \)   e) \( \text{ans} = 1 \ 2 \ 3 \ 4 \ 5 \)

   \( \text{linspace}(0,5,6) = [0 \ 1 \ 2 \ 3 \ 4 \ 5] \) and \( [0:5] = [0 \ 1 \ 2 \ 3 \ 4 \ 5] \)

   \( x=\text{linspace}(0,5,6) - [0:5] = [0 \ 1 \ 2 \ 3 \ 4 \ 5] - [0 \ 1 \ 2 \ 3 \ 4 \ 5] = [0 \ 0 \ 0 \ 0 \ 0 \ 0] \)

12. Executing the Matlab statements \( \theta=[0 \ \pi/2], \ x=\cos(\theta) - \sin(\theta) \), will result in
   
   a) an error message   b) \( x=[1.0000 \ -1.0000] \)   c) \( x=[0.5000 \ 1.0000] \)
   d) \( x=[-1.0000 \ 0] \)   e) \( x=[0 \ \ 0] \)

   \[
   x = \cos(\theta) - \sin(\theta)
   = \cos([0 \ \pi/2]) - \sin([0 \ \pi/2])
   = [\cos(0) \ \cos(\pi/2)] - [\sin(0) \ \sin(\pi/2)]
   = [1 \ 0] - [0 \ 1]
   = [1 \ -1]
   \]