Fa 96
EGN 3420 Exam 2 Name ________________

SHOW ALL WORK!

Problem 1 (25 pts)

Fit a saturation growth model to the data in the table.

x	1	2	2.5	4	6	8	8.5
y	0.4	0.7	0.8	1.0	1.2	1.3	1.4

Work Area

x	y	$u=\frac{1}{x}$	$z=\frac{1}{y}$
1	0.4	1	2.5
2	0.7	0.5	1.4286
2.5	0.8	0.4	1.25
4	1.0	0.25	1
6	1.2	0.1667	0.8333
8	1.3	0.125	0.7692
8.5	1.4	0.1176	0.7143

$y = a\frac{x}{b+x}$, $\frac{1}{y} = \frac{b}{a}\left(\frac{1}{x}\right) + \frac{1}{a}$

$z = a_1 \quad + a_0$

where $a_1 = \frac{b}{a}$ & $a_0 = \frac{1}{a}$

Applying linear regression to the data yields

$a_0 = 0.489$ & $a_1 = 1.9819$

$\Rightarrow a = \frac{1}{a_0} = 2.045$

$b = \frac{a_1}{a_0} = 4.053$

$$y = 2.045 \frac{x}{4.053 + x}$$

Ans. y = ________________

Fa 96

EGN 3420 Exam 2 Name ________________

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Problem 2 (25 pts)

Fit a 3rd order Newton Divided Difference Interpolating polynomial thru the data given below and estimate the error at x=3.5 by using an additional data point at (6,36).

x	1	2	3	5
f(x)	4.75	4	5.25	19.75

Work Area

i	x_i	$f(x_i)$	Δ	Δ^2	Δ^3	Δ^4
0	1	4.75	-0.75	1	0.25	0
1	2	4	1.25	2	0.25	
2	3	5.25	7.25	3		
3	5	19.75	16.25			
4	6	36				

$$f_3(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$= 4.75 - 0.75(x-1) + (x-1)(x-2) + 0.25(x-1)(x-2)(x-3)$$

$$f_3(3.5) = 4.75 - 0.75(2.5) + (2.5)(1.5) + 0.25(2.5)(1.5)(0.5)$$

$$= 7.09375$$

Estimate of error $= f_4(3.5) - f_3(3.5) = 0$ Since $b_4 = 0$

$f_3(x) =$ ________________________________

Estimate of error in $f_3(3.5) =$ ________

Fa 96
EGN 3420 Exam 2 Name ________________

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Problem 3 (25 pts)

Evaluate the integral $\int_0^{\pi} (4 + 2\sin x)\, dx$

a) Analytically
b) By trapezoidal integration using 8 intervals
c) By Simpson's 1/3 formula using 4 intervals

Work Area

a) $\int_0^{\pi} (4+2\sin x)\,dx = 4x - 2\cos x \Big|_0^{\pi} = 4\pi - 2\cos\pi + 2\cos 0 = 16.56637$

b) (n = 8)

i	x_i	$f(x_i)$
0	0	4.00000
1	$\pi/8$	4.76537
2	$\pi/4$	5.41421
3	$3\pi/8$	5.84776
4	$\pi/2$	6.00000
5	$5\pi/8$	5.84776
6	$3\pi/4$	5.41421
7	$7\pi/8$	4.76537
8	π	4.00000

$$I_T = \frac{b-a}{n}\left[\frac{f(x_0)}{2} + f(x_1) + \dots + f(x_7) + \frac{f(x_8)}{2}\right]$$

$= 16.51483$

c) (n = 4)

i	x_i	$f(x_i)$
0	0	4.00000
1	$\pi/4$	5.41421
2	$\pi/2$	6.00000
3	$3\pi/4$	5.41421
4	π	4.00000

$$I_S = \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$= 16.57548$

Ans. a) 16.56637 b) 16.51483 c) 16.57548

Fa 96
EGN 3420 Exam 2 Name ________________

SHOW ALL WORK!

Problem 4 (25 pts)

Use Richardson's Extrapolation with 4 and 8 intervals to obtain an estimate of

$$I = \int_0^4 xe^{2x}\,dx$$

with truncation error $O(h^4)$.

Work Area

i	x_i	$f(x_i)$
0	0	0
1	0.5	1.35914
2	1	7.38906
3	1.5	30.12831
4	2	109.19630
5	2.5	371.03289
6	3	1210.28638
7	3.5	3838.21605
8	4	11923.83195

$n=4,\ x_0=0,\quad f_0=0$
$x_1=1,\quad f_1=7.38906$
$x_2=2,\quad f_2=109.19630$
$x_3=3,\quad f_3=1210.28638$
$x_4=4,\quad f_4=11923.83195$

$$I_4=\frac{b-a}{n}\left(\frac{f_0}{2}+f_1+f_2+f_3+\frac{f_4}{2}\right)$$
$$=\frac{4-0}{4}\left(\frac{0}{2}+7.38906+\cdots+\frac{11923.83195}{2}\right)$$
$$=7288.78771$$

$m=8,\quad I_8=\frac{b-a}{m}\left(\frac{f_0}{2}+f_1+f_2+f_3+f_4+f_5+f_6+f_7+\frac{f_8}{2}\right)$

$$=\frac{4-0}{8}\left(\frac{0}{2}+1.35914+\cdots+3838.21605+\frac{11923.83195}{2}\right)$$
$$=5764.76205$$

$$I=I_n+\frac{I_n-I_m}{(k/h)^2-1}=I_4+\frac{I_4-I_8}{(0.5/1)^2-1}=7288.78771+\frac{7288.78771-5764.76205}{-0.75}$$

Ans. $I=5256.75350$