EGN 3420 EXAM I Name________________

Fa 96

SHOW ALL WORK!

Problem 1 (25 pts)

Consider the function $f(x) = e^x + x^2$. A third order truncated Taylor Series of f(x) expanded about the point $x_0 = a$ and evaluated at the point x=2a is given below.

$$f_3(2a) = e^a (\alpha_0 + \alpha_1 a + \alpha_2 a^2 + \alpha_3 a^3) + \beta a^2$$

A) Find α_0, α_1, α_2, α_3, and β.

B) For a = 0.1, find the truncation error $E_T = f(0.2) - f_3(0.2)$ to 6 places after the decimal point.

Work Area

A) $f(x) = e^x + x^2$, $f_3(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)(x-x_0)^3}{3!}$

$f'(x) = e^x + 2x$, $f_3(x) = (e^a + a^2) + (e^a + 2a)(x-a) + \frac{(e^a+2)}{2}(x-a)^2 + \frac{e^a}{6}(x-a)^3$

$f''(x) = e^x + 2$,

$f'''(x) = e^x$, $f_3(2a) = (e^a + a^2) + (e^a + 2a)a + \frac{(e^a+2)}{2}a^2 + \left(\frac{e^a}{6}\right)a^3$

$f_3(2a) = e^a\left(1 + a + \frac{1}{2}a^2 + \frac{1}{6}a^3\right) + 4a^2$

$\Rightarrow \alpha_0 = 1,\ \alpha_1 = 1,\ \alpha_2 = \frac{1}{2},\ \alpha_3 = \frac{1}{6},\ \beta = 4$

B) $f(0.2) = e^{0.2} + (0.2)^2 = 1.261403$

$f_3(0.2) = e^{0.1}\left(1 + 0.1 + \frac{1}{2}0.1^2 + \frac{1}{6}0.1^3\right) + 4(0.1)^2 = 1.261398$

$E_T = f(0.2) - f_3(0.2) = 1.261403 - 1.261398 = 0.000005$

....

Ans. α_0 = 1, α_1 = 1, α_2 = 1/2, α_3 = 1/6, β = 4

E_T = 0.000005

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Problem 2 (30 pts)

The function $f(x) = x^4 - 5x^2 + 4$ has a root located between 0 and 1.5. Fill in the tables below for the first three iterations of the Bisection Method and the False Position method. Express all answers to four digits after the decimal point.

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_r)$	e_A
1	0.0000	1.5000	0.7500	4.0000	1.5039	-
2	0.7500	1.5000	1.1250	1.5039	-0.7263	0.3333
3	0.7500	1.1250	0.9375	1.5039	0.3779	-0.2000

Bisection Method

$$x_r = \frac{x_l + x_u}{2}$$

Iteration	x_l	x_u	x_r	$f(x_l)$	$f(x_u)$	$f(x_r)$	e_A
1	0.0000	1.5000	0.9697	4.0000	-2.1875	0.1826	-
2	0.9697	1.5000	1.0106	0.1826	-2.1875	-0.0632	0.0405
3	0.9697	1.0106	1.0001	0.1826	-0.0632	-0.0006	-0.0105

False Position Method

$$x_r = x_l - f(x_l) \left\{ \frac{x_u - x_l}{f(x_u) - f(x_l)} \right\}$$

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Problem 3 (20 pts)

Use the one point iteration method to locate the smaller root of the quadratic

$f(x) = x^2 - 10x + 1$. Choose $g(x) = (x^2 + 1)/10$.

Stop when the magnitude of the true error, $|E_T|$ falls below 0.00001

i	x_i	$x_{i+1} = g(x_i)$	$\lvert E_T \rvert = \lvert \text{True root} - x_i \rvert$
0	5.00000	2.60000	4.89898
1	2.6000	0.77600	~~0.67498~~ 2.49898
2	0.77600	0.16022	~~0.0592~~0 0.67498
3	0.16022	0.10257	~~0.00155~~ 0.0592
4	0.10257	0.10105	~~0.000003~~ 0.00155
5	0.10105	0.10102	0.0000031
6	0.10102		0.0000000
7			
8			

$$\text{True Root} = \frac{10 \pm \sqrt{100 - 4(1)(1)}}{2}$$

$$= 0.10102, \; 9.89898$$

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Problem 4 (25 pts)

Find the matrix inverse A^{-1} to solve the system of equations $A\underline{x} = \underline{b}$ given below.

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6 \\ x & - & 2y & + & z & = & 0 \\ 4x & + & y & - & 2z & = & 0 \end{array}$$

Work Area

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 4 & 1 & -2 \end{pmatrix} \qquad |A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -2 & 1 \\ 4 & 1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -3 & 0 \\ 0 & -3 & -6 \end{vmatrix} = 18$$

$$\text{Cof } A = \begin{pmatrix} 3 & 6 & 9 \\ 3 & -6 & 3 \\ 3 & 0 & -3 \end{pmatrix}, \quad \text{Adj } A = \begin{pmatrix} 3 & 3 & 3 \\ 6 & -6 & 0 \\ 9 & 3 & -3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{18} \begin{pmatrix} 3 & 3 & 3 \\ 6 & -6 & 0 \\ 9 & 3 & -3 \end{pmatrix} = \frac{1}{6} \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 3 & 1 & -1 \end{pmatrix}$$

$$\underline{x} = A^{-1} \underline{b} = \frac{1}{6} \begin{pmatrix} 1 & 1 & 1 \\ 2 & -2 & 0 \\ 3 & 1 & -1 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Ans. $A^{-1} = \begin{bmatrix} \quad \end{bmatrix}$, x = ____, y = ____, z = ____