Fa 94

EGN 3420 FINAL Name ___________

Problem 1 (20 pts)

1. **Given the system of equations below, find the solution $\underline{x} = A^{-1}\underline{b}$. Use either method discussed in class to find the inverse A^{-1}.**

$$
\begin{array}{ccccccc}
x & + & y & + & z & = & 0 \\
3x & - & y & + & 2z & = & 7 \\
4x & + & y & - & 2z & = & 0
\end{array}
$$

....

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Problem 2 (15 pts)

Find the value(s) of K for which the equations below have a unique solution.

$$\begin{aligned} x + y + 2z &= K \\ 3x - y + 2z &= 7 \\ 4x + Ky - 2z &= 0 \end{aligned}$$

....

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Problem 3 (15 pts)

Find the value(s) of K for which the solution to the system of equations below is x=1, y=-2, z=1.

$$
\begin{aligned}
x &+ y &+ z &= K \\
3x &- (1+K)y &+ 2z &= 7 \\
4x &+ y &- 2z &= K
\end{aligned}
$$

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Problem 4 (20 pts)

In the system of equations below, are there any arbitrary unknowns? If yes, without solving for any of the variables u, x, y, or z, determine if y or z can be arbitrary. Show all work.

$$
\begin{array}{ccccccccccc}
u & + & x & & & + & z & = & 3 \\
u & - & x & + & 2y & & & = & 0 \\
 & & x & - & y & - & 3z & = & -2 \\
u & & & + & y & + & z & = & 2
\end{array}
$$

Work Area

(1) $A\underline{x} = \underline{b}$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 4 & 1 & -2 \end{pmatrix}, \quad \underline{b} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 4 & 1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & -4 & -1 \\ 0 & -3 & -6 \end{vmatrix} = 24 - 3 = 21$$

$$\mathrm{Cof}(A) = \begin{pmatrix} 0 & 14 & 7 \\ 3 & -6 & 3 \\ 3 & 1 & -4 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \mathrm{Adj}A = \frac{1}{21} \begin{pmatrix} 0 & 3 & 3 \\ 14 & -6 & 1 \\ 7 & 3 & -4 \end{pmatrix}$$

$$\underline{x} = A^{-1}\underline{b}$$

$$= \frac{1}{21} \begin{pmatrix} 0 & 3 & 3 \\ 14 & -6 & 1 \\ 7 & 3 & -4 \end{pmatrix} \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

②

$$x + y + 2z = k$$
$$3x - y + 2z = 7$$
$$4x + ky - 2z = 0$$

$$A\underline{x} = \underline{b}$$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 3 & -1 & 2 \\ 4 & k & -2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ 0 & -4 & -4 \\ 0 & k-4 & -10 \end{vmatrix} = 40 - (-4)(k-4) = 4k+24$$

For unique solution, $|A| \neq 0$

$$\Rightarrow 4k + 24 \neq 0$$
$$k \neq -6$$

③

$$x + y + z = k$$
$$3x - (1+k)y + 2z = 7$$
$$4x + y - 2z = k$$

Solution: $x = 1,\ y = -2,\ z = 1$

$\Rightarrow \quad 1 + (-2) + 1 = k \qquad$ From 1st equation.

$$0 = k$$

Check

$$3(1) - (1+0)(-2) + 2(1) = 7 \ \checkmark$$
$$4(1) + (-2) - 2(1) = 0 \ \checkmark$$

(4).

$$\begin{aligned}
u + x \qquad\quad + z &= 3 \\
u - x + 2y \qquad\quad &= 0 \\
x - y - 3z &= -2 \\
u \qquad + y + z &= 2
\end{aligned}$$

$$(A \,\vdots\, \underline{b}) = \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 3 \\ 1 & -1 & 2 & 0 & 0 \\ 0 & 1 & -1 & -3 & -2 \\ 1 & 0 & 1 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 3 \\ 0 & -2 & 2 & -1 & -3 \\ 0 & 1 & -1 & -3 & -2 \\ 0 & -1 & 1 & 0 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -3 & -2 \\ 0 & 0 & 0 & -7 & -7 \\ 0 & 0 & 0 & -3 & -3 \end{array}\right) \sim \begin{array}{c} \begin{array}{cccc} u & x & y & z \end{array} \\ \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -3 & -2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \end{array}$$

There is one arbitrary unknown.

y is arbitrary if $\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{vmatrix} \neq 0$

Since the determinant above equals 1, y is arbitrary.

z is arbitrary if $\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{vmatrix} \neq 0$

Since the determinant above equals 0, z is not arbitrary.

(5)

i	x_i	$f_i = f(x_i) = \frac{1}{1+x_i}$
0	0	1.00000
1	0.125	0.88889
2	0.250	0.80000
3	0.375	0.72727
4	0.500	0.66667
5	0.625	0.61538
6	0.750	0.57143
7	0.875	0.53333
8	1	0.50000

A) $$I_4 = h\left[\frac{f_0+f_4}{2} + f_1+f_2+f_3\right], \quad h = 0.25$$

$$= 0.25\left[\frac{1+0.5}{2} + 0.8 + 0.66667 + 0.57143\right]$$

$$= 0.69702$$

B) $$I_8 = h\left[\frac{f_0+f_8}{2} + f_1+f_2+f_3+f_4+f_5+f_6+f_7\right], \quad h = 0.125$$

$$= 0.125\left[\frac{1+0.5}{2} + 0.88889 + 0.80000 + \cdots + 0.53333\right]$$

$$= 0.69412$$

c) $$I_{4/8} = I_4 + \frac{I_4 - I_8}{\left(\frac{0.125}{0.250}\right)^2 - 1} = 0.69702 + \frac{0.69702 - 0.69412}{(1/2)^2 - 1}$$

$$= 0.69315$$

D) $I_8 = \frac{h}{3}(f_0 + 4f_1 + 2f_2 + \cdots + 2f_6 + 4f_7 + f_8)$

$= \frac{h}{3}\left[f_0 + 4(f_1 + f_3 + f_5 + f_7) + 2(f_2 + f_4 + f_6) + f_8 \right]$

$= \frac{0.125}{3}\left[1 + 4(0.88889 + \cdots + 0.53333) + 2(0.8 + \cdots + 0.57143) + 0.5 \right]$

$= 0.69315$

E) Global Truncation Error $= I - I_8$

$= \ln 2 - 0.69315$

$= 0.69315 - 0.69315$

$= 0$