

(1) Find the inverse of the matrix A using the method of cofactors.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Ans. $A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

2) Use the inverse to find the solution to the system of equations:

$$\begin{array}{rclclcl} x_1 & - & x_2 & + & x_3 & = & 2/3 \\ -3x_1 & + & 2x_2 & + & x_3 & = & 4/3 \\ 2x_1 & + & x_2 & + & 3x_3 & = & 13/3 \end{array}$$

Ans. $x_1 = \underline{\hspace{2cm}}$ $x_2 = \underline{\hspace{2cm}}$ $x_3 = \underline{\hspace{2cm}}$

Solve

$$\begin{aligned}x_1 - x_2 + x_3 &= 2/3 \\ -3x_1 + 2x_2 + x_3 &= 4/3 \\ 2x_1 + x_2 + 3x_3 &= 13/3\end{aligned}$$

$$Ax = b$$

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix}, \quad b = \frac{1}{3} \begin{pmatrix} 2 \\ 4 \\ 13 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ -3 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 1 \\ 0 & -1 & 4 \\ 0 & 3 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 4 \\ 3 & 1 \end{vmatrix} = -13$$

$$A^c = \begin{pmatrix} 5 & 11 & -7 \\ 4 & 1 & -3 \\ -3 & -4 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{-13} \begin{pmatrix} 5 & 4 & -3 \\ 11 & 1 & -4 \\ -7 & -3 & -1 \end{pmatrix}$$

check:

$$\frac{1}{-13} \begin{pmatrix} 5 & 4 & -3 \\ 11 & 1 & -4 \\ -7 & -3 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & 1 \\ 2 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \checkmark$$

$$\begin{aligned}\underline{x} &= A^{-1} \underline{b} = \frac{-1}{13} \begin{pmatrix} 5 & 4 & -3 \\ 11 & 1 & -4 \\ -7 & -3 & -1 \end{pmatrix} \frac{1}{3} \begin{pmatrix} 2 \\ 4 \\ 13 \end{pmatrix} \\ &= \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix}\end{aligned}$$