

Fa 94
EGN 3420

Exam
QUIZ 2

Name _____

(2) Given the following equations:

$$\begin{array}{rcccccccl}
 w & + & x & + & y & & & = & K \\
 & & 2x & - & y & - & z & = & -1 \\
 2w & + & x & + & y & - & z & = & 3 \\
 3w & - & 4x & & & - & 3z & = & 3
 \end{array}$$

1. Find the value of K for which the equations are consistent.
2. Is it possible to find a unique solution? Explain.
3. Without solving for w, x, y, or z show that when the equations are consistent, x cannot be arbitrary.

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WORK AREA

$$w = 1 + z$$

$$x = 0$$

$$y = 1 - z$$

$$z = \text{arbitrary}$$

$$w + x + y + z = 2 + z$$

$$2x - y - z = -1$$

$$2w + x + y + z = 3 + z$$

$$3w - 4x + z = 3 + z$$

$$-3z$$

$$w + x + y = 2 \rightarrow K$$

$$2x - y - z = -1$$

$$2w + x + y - z = 3$$

$$3w - 4x - 3z = 3$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & K \\ 0 & 2 & -1 & -1 & -1 \\ 2 & 1 & 1 & -1 & 3 \\ 3 & -4 & 0 & -3 & 3 \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & K \\ 0 & 2 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 & 3-2K \\ 0 & -7 & -3 & -3 & 3-3K \end{array} \right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & K \\ 0 & 1 & 1 & 1 & -3+2K \\ 0 & 2 & -1 & -1 & -1 \\ 0 & -7 & -3 & -3 & 3-3K \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 0 & K \\ 0 & 1 & 1 & 1 & -3+2K \\ 0 & 0 & -3 & -3 & 5-4K \\ 0 & 0 & 4 & 4 & -18+11K \end{array} \right)$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & K \\ 0 & 1 & 1 & 1 & -3+2K \\ 0 & 0 & 1 & 1 & (-5+4K)/3 \\ 0 & 0 & 4 & 4 & -18+11K \end{array} \right) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & K \\ 0 & 1 & 1 & 1 & -3+2K \\ 0 & 0 & 1 & 1 & (-5+4K)/3 \\ 0 & 0 & 0 & 0 & -\frac{4}{3}(-5+4K) - 18 + 11K \end{array} \right)$$

f) There will be an infinite number of solutions if ;

$$\frac{-4}{3}(-5+4K) - 18 + 11K = 0$$

$$20 - 16K - 54 + 33K = 0$$

$$17K = 34$$

$$K = 2$$

Because $|A| = 0$

B) For $K=2$, $Ax = b$ becomes

$$\begin{pmatrix} w & x & y & z & & \\ 1 & 1 & 1 & 1 & | & 2 \\ 0 & 1 & 1 & 1 & | & 1 \\ 0 & 0 & 1 & 1 & | & 1 \end{pmatrix}$$

To see if x is arbitrary, write eqs as $\hat{A}\hat{x} = \hat{b}$, $\hat{x} = \begin{pmatrix} w \\ y \\ z \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} w & y & z & & | & \\ 1 & 1 & 1 & & | & 2-x \\ 0 & 1 & 1 & & | & 1-x \\ 0 & 1 & 1 & & | & 1 \end{pmatrix}$$

Since $|\hat{A}| = 0$, x is not arbitrary.