Assignment #5 Key; Due February 20 at start of class

1. Consider the set of indices \text{NonConstant} = NC = \{ f \mid |\text{range}(\varphi_f)| > 1 \}. Use Rice’s Theorem to show that NC is not recursive (not decidable). Note that members of NC do not need to converge for all input, but they must converge on at least two input values that produce different output values. Hint: There are two properties that must be demonstrated.

\text{First, NC is non-trivial as } I(x) = x \text{ is in NC and } Z(x) = 0 \text{ is not.}

\text{Second, NC is an I/O Property. To see this, let } f \text{ and } g \text{ be arbitrary indices of computable functions such that } \forall x \varphi_f(x) = \varphi_g(x).

f \text{ is in NC iff } |\text{range}(\varphi_f)| > 1. \text{ But } g \text{’s range is exactly that of } f \text{ and so, } 

|\text{range}(\varphi_f)| > 1 \text{ iff } |\text{range}(\varphi_g)| > 1. \text{ But then, } 

f \text{ is in NC iff } g \text{ is in NC}

Since NC is not trivial and is an I/O property then it is not recursive by Rice’s Theorem.

2. Show that \( K \preceq_m \text{NonConstant} \), where \( K = \{ f \mid \varphi_f(f) \downarrow \} \).

Let \( f \) be an arbitrary index of some computable function.

Then, \( f \) is in \( K \) iff \( \varphi_f(f) \downarrow \)

Define \( g_f(x) = \varphi_f(f) - \varphi_f(f) + x \)

For all \( x \), \( g_f(x) = x \) iff \( \varphi_f(f) \downarrow \); and diverges otherwise.

But then \( f \) is in \( K \) iff \( g_f \) is in NC.

This shows that \( K \preceq_m \text{NonConstant} \), as was desired.