Assignment#2; Due January 28 at start of class

Let set $A$ be non-empty recursive, $B$ be re non-recursive and $C$ be non-re. Using the terminology (REC) recursive, (RE) non-recursive recursively enumerable, (NR) non-re, categorize each set below, saying whether or not the set can be of the given category and justifying each answer. You may assume, for any set $S$, the existence of comparably hard sets $S_E = \{2x|x \in S\}$ and $S_D = \{2x+1|x \in S\}$. The following is a sample of the kind of answer I require:

Sample.) $A \cap C = \{ x | x \in A \text{ and } x \in C \}$

REC: Yes. If $A = \{0\}$ then $A \cap C = \emptyset$ or $\{0\}$, each of which is in REC.

RE: Yes. Let $A = \aleph_E = \{ 2x | x \in \aleph \}$; let $C = \text{TOT}_D \cup \text{HALT}_E$ then $A \cap C = \text{HALT}_E$ which is in RE.

NR: Yes. If $A = \aleph$ then $A \cap C = C$, which is in NR.

a.) $B - A = \{ x | x \in B \text{ and } x \notin A \}$ // Set difference

b.) $A * B = \{ x \cdot y | x \in A \text{ and } y \in B \}$ // Multiplication

c.) $A \cup C = \{ x | x \in A \text{ or } x \in C \}$ // Set union

Be careful: Some may not be possible. If so, you must justify why this is so.

Note:
TOT = $\{ x | \forall \phi_x (y) \downarrow \}$. These are the indices of the set of algorithms.
HALT = $\{ <x,y> | \phi_x (y) \downarrow \}$. This is the set of pairs of procedures and input for which the given procedure halts.

The set $S_E$ for any set $S$ is defined as $\{2x | x \in S\}$
$S_D = \{ 2x+1 | x \in S \}$. 