1. Consider the set of indices   
   DEFINED = { f | ∃x ϕf(x)↓ }.

Use Rice’s Theorem to show that DEFINED is not decidable.

Hint: There are two properties that must be demonstrated.

1. Let P = { f | ∃x ϕf(x) converges in at most x steps }. Why does Rice’s theorem not tell us anything about the undecidability of P?
2. Show that DEFINED is not decidable by reducing K0 to this set.
3. Is DEFINED re? Support your conclusion.
4. Let Incr = { f | ∀x ϕf(x+1)>ϕf(x) }.   
   Let TOT = { f | ∀x ϕf(x) converges }.   
   Prove that Incr ≡m TOT.
5. Let sets A and B each be re non-recursive.   
   Consider C = A ∩ B. For (a)-(c), either show sets A and B with the specified property or demonstrate that this property cannot hold.
   1. Can C be recursive?
   2. Can C be re non-recursive
   3. Can C be non-re?