Complexity Theory
More Complexity

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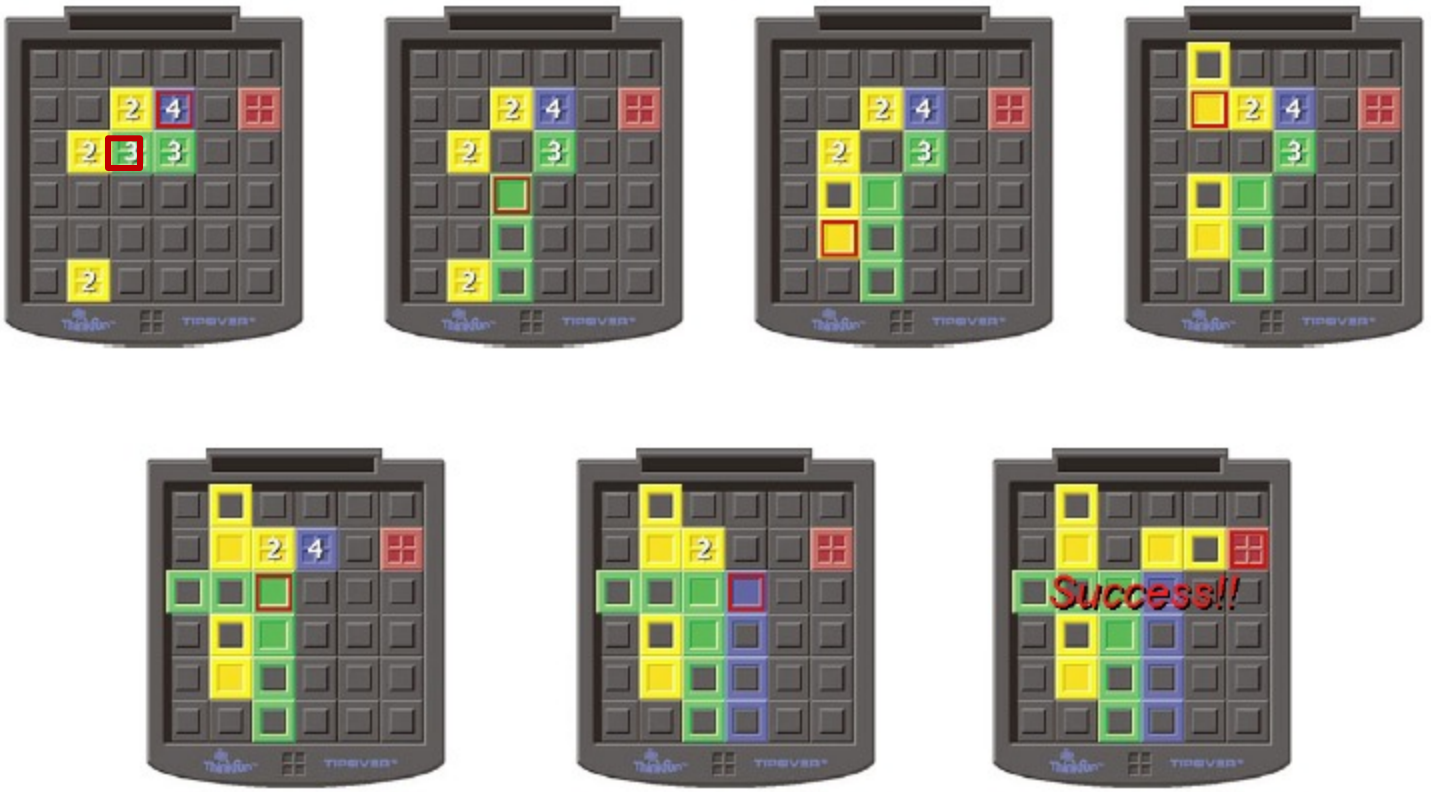
COT6410 – Spring 2023 Notes

More Examples of NP Complete Problems

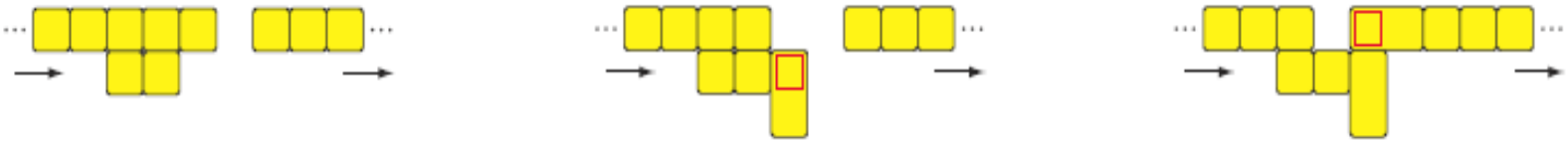
TipOver

Rules of Game



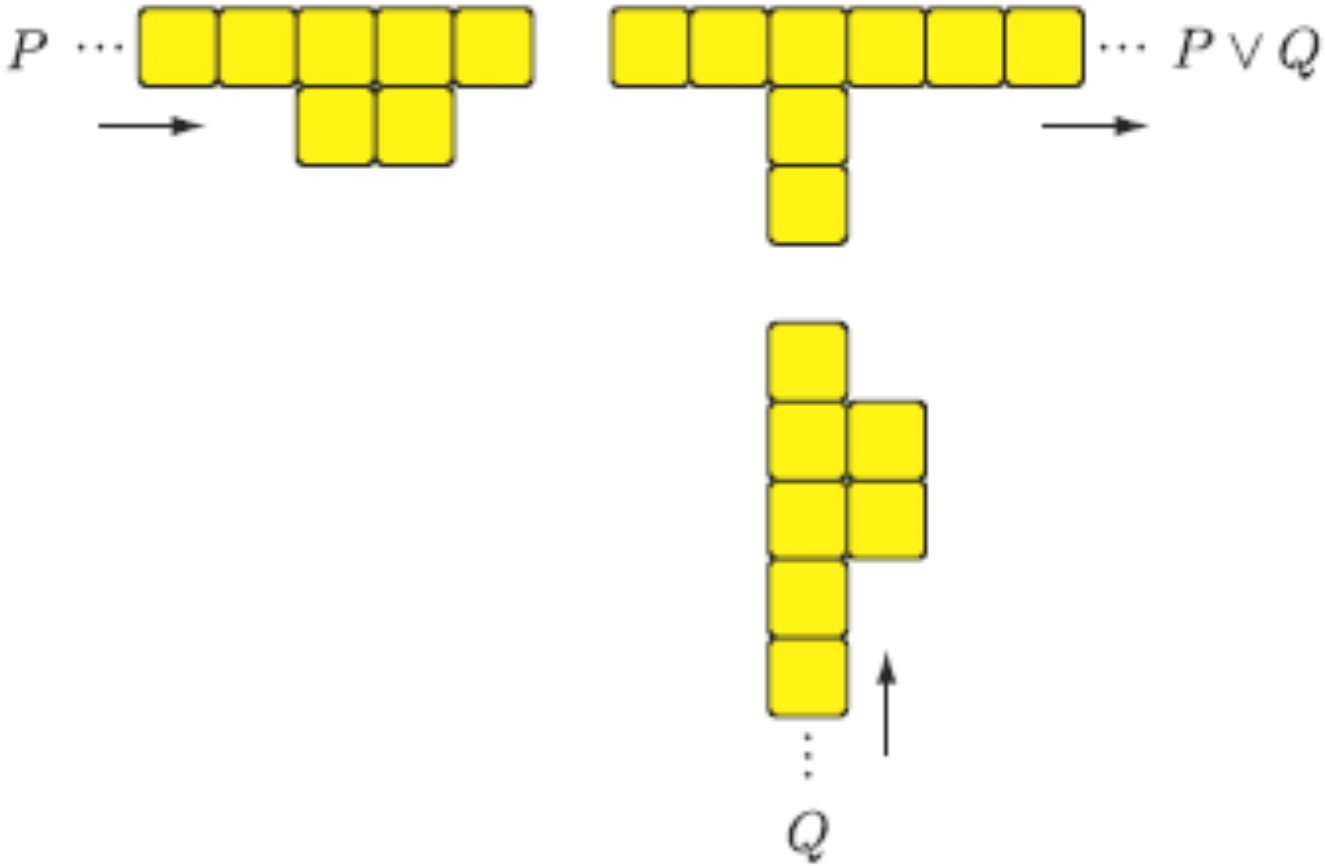
Numbers are height of crate stack;
If could get 4 high out of way we can attain goal

Directional gadget



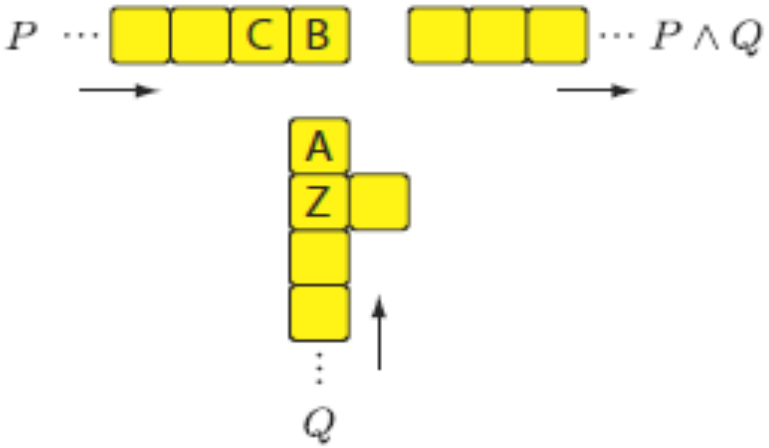
Single stack is two high;
tipped over stack is one high, two long;
red square is location of person travelling the towers
Note that there is a pathway back as well as forward

One directional Or gadget

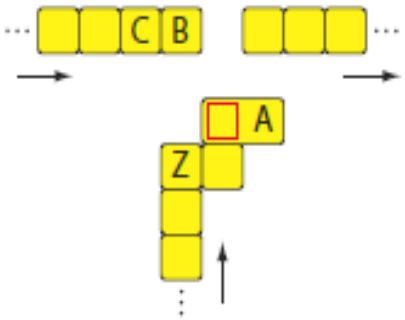


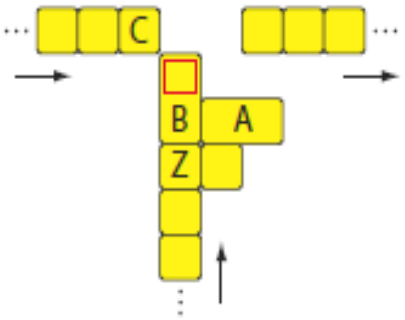
Note that there is a pathway back as well as forward

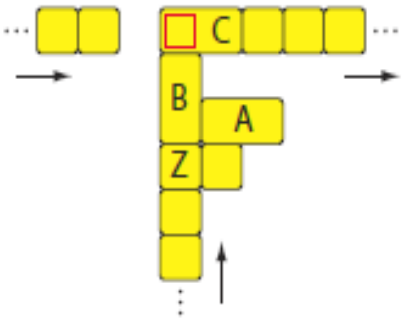
AND Gadget



How AND Works

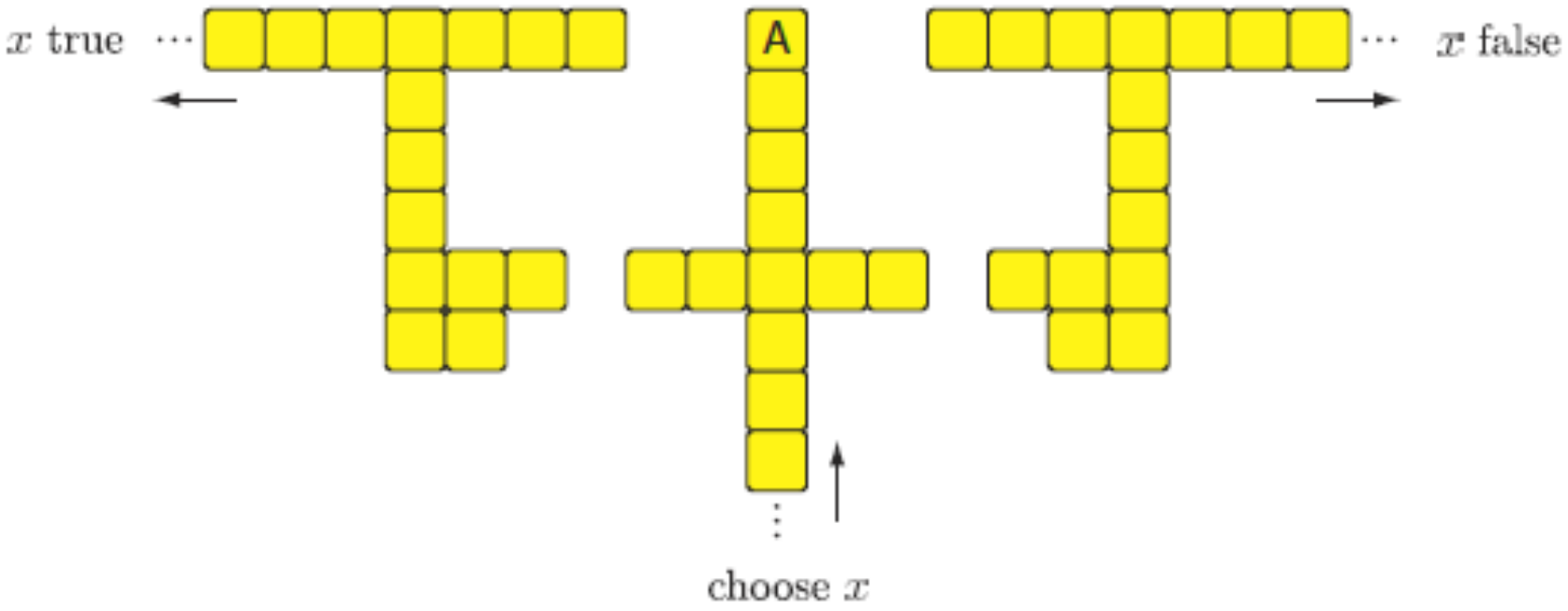






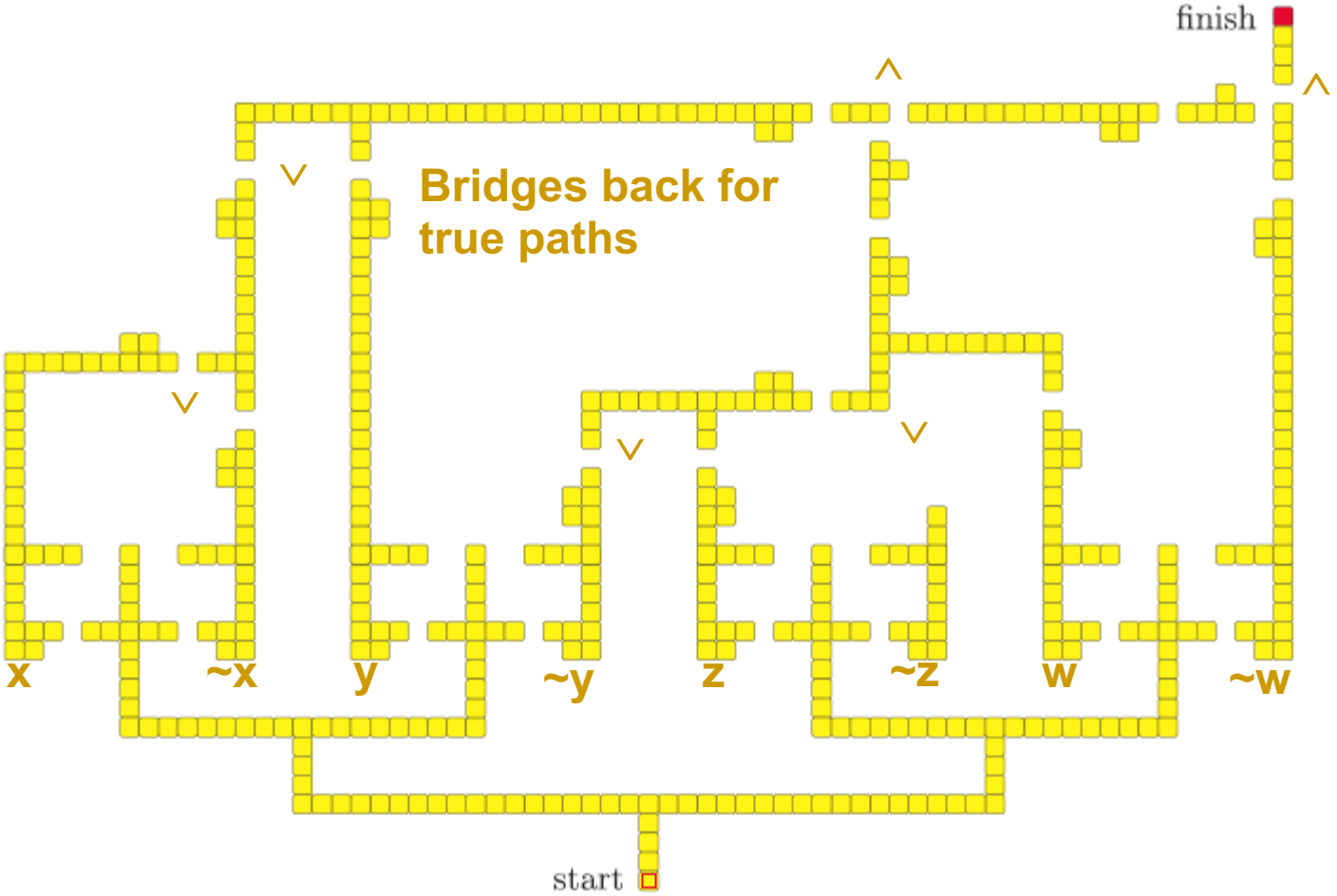
Note that there is a pathway back as well as forward

Variable Select Gadget



Tip A left to set x true; right to set x false
Can build bridge to go back but never to change choice

((x∨~x∨y)∧(~y∨z∨w)∧~w)



Win Strategy is NP-Complete

- **TipOver** win strategy is **NP-Complete**
- **Minesweeper** consistency is **NP-Complete**
- **Phutball** single move win is **NP-Complete**
  - Do not know complexity of general winning strategy
  - Determining from a fixed setup if a win is possible is **PSpace-Hard** (may not be in **PSpace**)
- **Checkers** is really interesting
  - Single move to **King** is in **P**
  - Winning strategy is **PSpace-Complete**