Assign#6 Key

Spring 2023

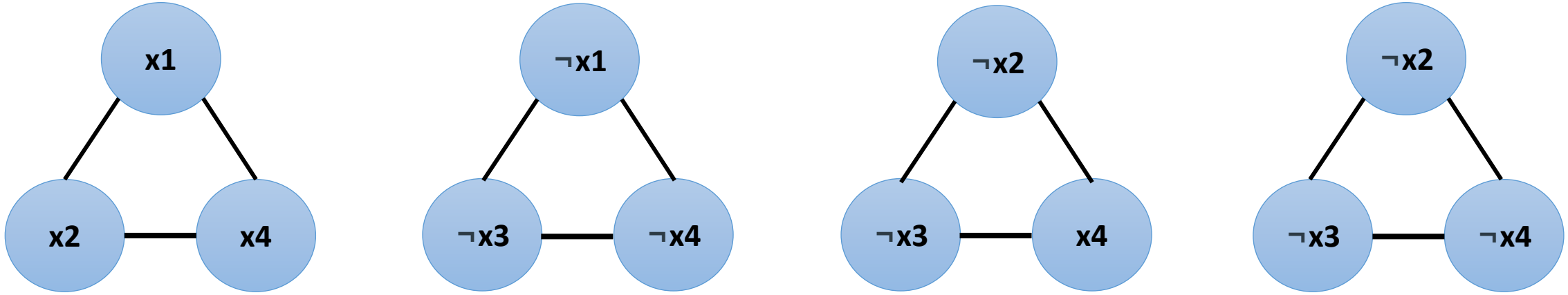
1. Consider the 3SAT instance:
   E = (x1 ∨ x2 ∨ x4) & (¬x1 ∨ ¬x3 ∨ ¬x4) & (¬x2 ∨ ¬x3 ∨ x4) & (¬x2 ∨ ¬x3 ∨ ¬x4)

Recast **E** as an instance of k-Independent Set and present a solution to the latter

Question 1 (a)

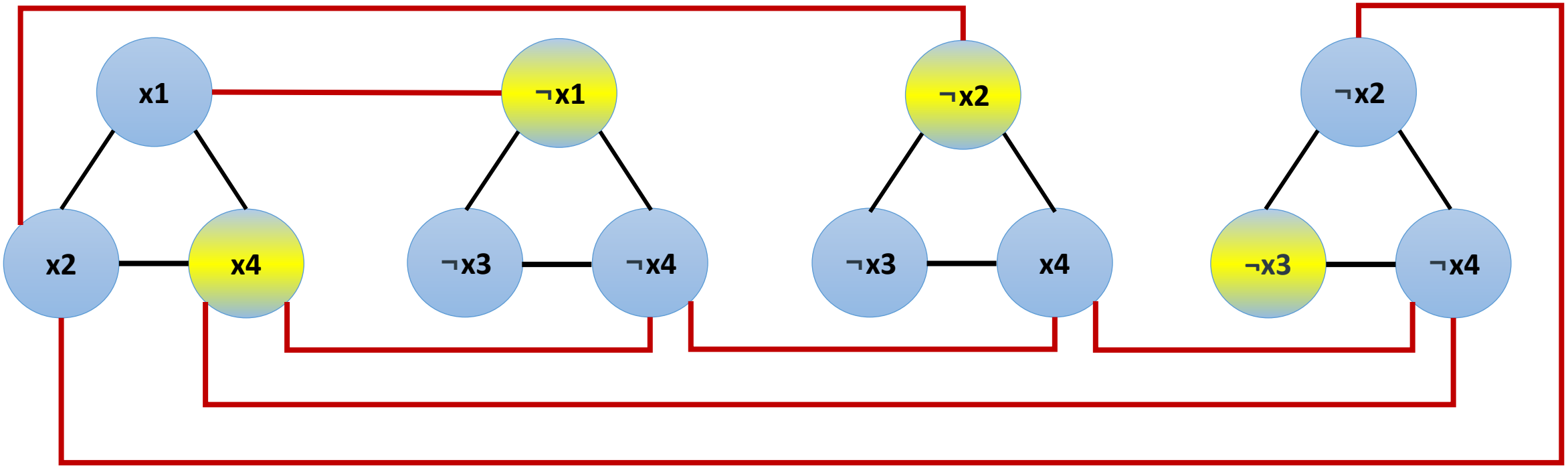
E = (x1 ∨ x2 ∨ x4) & (¬x1 ∨ ¬x3 ∨ ¬x4) & (¬x2 ∨ ¬x3 ∨ x4) & (¬x2 ∨ ¬x3 ∨ ¬x4)

Clause Gadgets:



E = (x1 ∨ x2 ∨ x4) & (¬x1 ∨ ¬x3 ∨ ¬x4) & (¬x2 ∨ ¬x3 ∨ x4) & (¬x2 ∨ ¬x3 ∨ ¬x4)

Combined Gadgets:

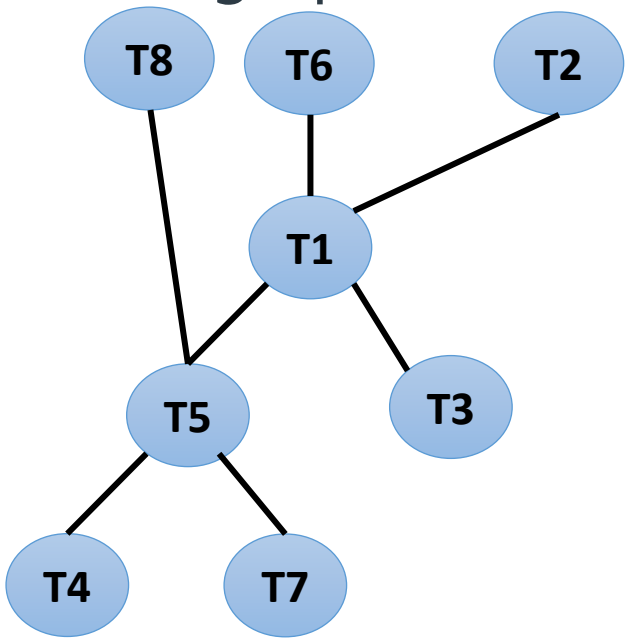


There are many solutions. I chose **¬x1 ¬x2 ¬x3 x4**

2. Task set **(T1,3), (T2,1), (T3,1), (T4,3), (T5,3), (T6,3), (T7,5), (T8,5)** with partial order

T1<T3; T1<T5, T2<T1, T5<T4; T5<T7; T6<T1, T8 < T3

a. Draw the graph that depicts these relationships.



b. Show the 2-processor schedule that results when the task number is the priority; a smaller task number means higher priority.

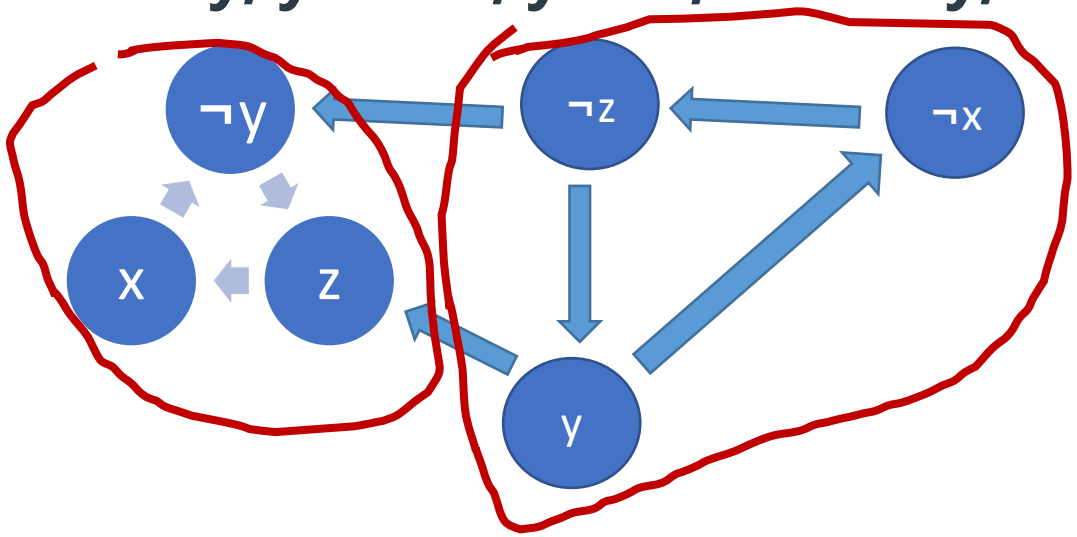
T2	T8	T8	T8	T8	T8	T5	T5	T5	T4	T4	T4								
T6	T6	T6	T1	T1	T1	T3			T7	T7	T7	T7	T7						

3. Consider the following 2SAT instance.

(¬x ∨ ¬y) (¬y ∨ z) (¬z ∨ x) (z ∨ y)

a. Draw the implication graph associated with this formula.

x → ¬y; y → ¬x; y → z; ¬z → ¬y; z → x; ¬x → ¬z; ¬z → y; ¬y → z

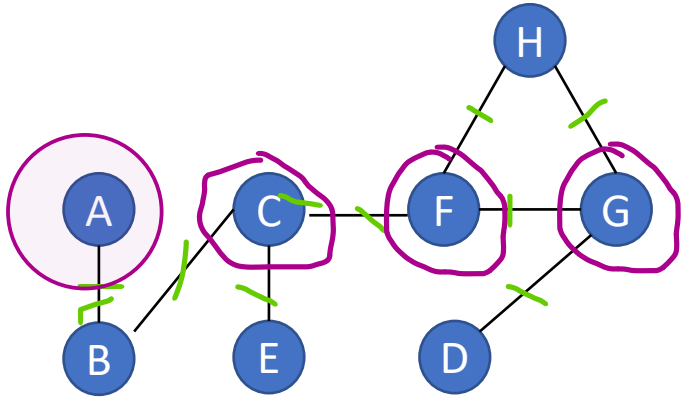


b. Draw circles around the strongly connected components (see red circles)

c. Provide a solution based on the SCCs or highlight the conflict exposed by the SCCs – the left cluster with three elements has no outgoing edges, so

x = ¬y = z = T

4. Consider the following instance of Positive Min-Ones-2SAT,
(A ∨ B) (B ∨ C) (C ∨ E) (C ∨ F) (D ∨ G) (F ∨ G) (F ∨ H) (G ∨ H)
a. Convert this instance of Positive 2SAT to a graph for which Min Vertex Cover is equivalent to the Min-Ones problem.



b. Show solution for Min Vertex Cover for (a) and correspondingly for the Positive Min-Ones-2SAT instance.

Solution: Min Cover is 4 choosing **A, C, F, G**; True assignments are **A = C = F = G = T**

See circled nodes in red and covered edges with green slashes.