Assign#6 Key

Spring 2023
1. Consider the 3SAT instance:

$$E = (x_1 \lor x_2 \lor x_4) \land (\neg x_1 \lor \neg x_3 \lor \neg x_4) \land (\neg x_2 \lor \neg x_3 \lor x_4) \land (\neg x_2 \lor \neg x_3 \lor \neg x_4)$$

Recast $E$ as an instance of $k$-Independent Set and present a solution to the latter.
Question 1 (a)

\[ E = (x_1 \lor x_2 \lor x_4) \land (\neg x_1 \lor \neg x_3 \lor \neg x_4) \land (\neg x_2 \lor \neg x_3 \lor x_4) \land (\neg x_2 \lor \neg x_3 \lor \neg x_4) \]

Clause Gadgets:
\[ E = (x_1 \lor x_2 \lor x_4) \land (\neg x_1 \lor \neg x_3 \lor \neg x_4) \land (\neg x_2 \lor \neg x_3 \lor x_4) \land (\neg x_2 \lor \neg x_3 \lor \neg x_4) \]

**Combined Gadgets:**

There are many solutions. I chose \( \neg x_1 \neg x_2 \neg x_3 \ x_4 \)
2. Task set \((T1,3), (T2,1), (T3,1), (T4,3), (T5,3), (T6,3), (T7,5), (T8,5)\) with partial order \(T1 < T3; T1 < T5, T2 < T1, T5 < T4; T5 < T7; T6 < T1, T8 < T3\)

a. Draw the graph that depicts these relationships.

b. Show the 2-processor schedule that results when the task number is the priority; a smaller task number means higher priority.
3. Consider the following 2SAT instance.
\((\neg x \lor \neg y) (\neg y \lor z) (\neg z \lor x) (z \lor y)\)

a. Draw the implication graph associated with this formula.
\(x \rightarrow \neg y; \ y \rightarrow \neg x; \ y \rightarrow z; \ \neg z \rightarrow \neg y; \ z \rightarrow x; \ \neg x \rightarrow \neg z; \ \neg z \rightarrow y; \ y \rightarrow z\)

b. Draw circles around the strongly connected components (see red circles)

c. Provide a solution based on the SCCs or highlight the conflict exposed by the SCCs – the left cluster with three elements has no outgoing edges, so \(x = \neg y = z = T\)
4. Consider the following instance of Positive \text{Min-Ones}-2\text{SAT},
\[(A \lor B) \land (B \lor C) \land (C \lor E) \land (C \lor F) \land (D \lor G) \land (F \lor G) \land (F \lor H) \land (G \lor H)\]
a. Convert this instance of Positive 2\text{SAT} to a graph for which \text{Min Vertex Cover} is equivalent to the \text{Min-Ones} problem.

b. Show solution for \text{Min Vertex Cover} for (a) and correspondingly for the Positive \text{Min-Ones}-2\text{SAT} instance.

Solution: \text{Min Cover} is 4 choosing \(A, C, F, G\); True assignments are \(A = C = F = G = T\)

See circled nodes in red and covered edges with green slashes.