Assignment#4 Key
1. Define \( \text{PredicateLike}(\text{PL}) = \{ f \mid \text{for each } x, f(x) \text{ either diverges or evaluates to 0 or 1} \} \).

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of \( \text{PL} \).

\[ \forall <x,t. (\text{STP}(f, x, t) \Rightarrow (\text{VALUE}(f, x, t) = 0) \lor \text{VALUE}(f, x, t) = 1)) \]

b.) Use Rice’s Theorem to prove that \( \text{PL} \) is undecidable. Be Complete.

\( \text{PL} \) is non-trivial as \( C_0(x) = 0 \in \text{PL} \) and \( S(x) = x + 1 \notin \text{PL} \)

Let \( f, g \) be two arbitrary indices of procedures such that \( \forall x \ f(x) = g(x) \)

\[ f \in \text{PL} \iff \forall x.(f(x) \uparrow \mid f(x) = 0 \mid f(x) = 1) \]

\[ \iff \forall x.(g(x) \uparrow \mid g(x) = 0 \mid g(x) = 1) \text{ since } \forall x f(x) = g(x) \]

\[ \iff g \in \text{PL} \]
1. Define \( \text{PredicateLike}(PL) = \{ f \mid \text{for each } x, f(x) \text{ either diverges or evaluates to 0 or 1} \} \).

c.) Show that \( \text{Diverge} = \{ <f,x> \mid f(x) \uparrow \} \) is many-one reducible to \( PL \).

Let \( f \) be an arbitrary index and input value.

From \( <f,x> \), define \( \forall y \ F_{f,x}(y) = f(x) - f(x) + 2 \)
\( <f,x> \in \text{Diverge} \Rightarrow f(x) \uparrow \Rightarrow \forall y \ F_f(y) \uparrow \Rightarrow F_f \in PL \)
\( <f,x> \notin \text{Diverge} \Rightarrow \forall y \ F_f(y) = 2 \Rightarrow F_f \notin PL \)

\( \text{Diverge} \leq_m \text{PL} \)

d.) Show that \( PL \) is many-one reducible to \( \text{Diverge} = \{ <f,x> \mid f(x) \uparrow \} \).

Let \( f \) be an arbitrary index.

From \( f \), define \( \forall x \ F_f(x) = \exists y [f(y) \neq 0 \& f(y) \neq 1] \)
\( f \in PL \Rightarrow \forall x (f(x) \uparrow | f(x) = 0 | f(x) = 1) \Rightarrow \forall x F_f(x) \uparrow \Rightarrow F_f \in \text{Diverge} \)
\( f \notin PL \Rightarrow \exists x (f(x) \downarrow \& f(x) \neq 0 \& f(x) \neq 1) \Rightarrow \exists x F_f(x) \downarrow \Rightarrow F_f \notin \text{Diverge} \)

\( PL \leq_m \text{Diverge} \)
2. Define IsPredicate(IP) = { f | for each x, f(x) evaluates to 0 or 1 }.

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of IP

∀x∃t [STP(f, x, t) & (VALUE(f, x, t) = 0 | VALUE(f, x, t) = 1)]

b.) Use Rice’s Theorem to prove that IP is undecidable. Be Complete.

IP is non-trivial as C₀(x) = 0 ∈ IP and S(x) = x+1 ∉ IP

Let f, g be two arbitrary indices of procedures such that ∀x f(x) = g(x).

f ∈ IP ⇔ ∀x, f(x) = 1 | f(x) = 0
⇔ ∀x, g(x) = 1 | g(x) = 0 since ∀xf(x) = g(x)
⇔ g ∈ IP
2. Define IsPredicate(IP) = \{ f \mid \forall x, f(x) \text{ evaluates to 0 or 1} \}.

c.) Show that TOTAL = \{ f \mid \forall x f(x)\downarrow \} is many-one reducible to IP.

Let f be an arbitrary index and input value.

From f, define F^f(x) = f(x) - f(x).

f \in TOTAL \Rightarrow \forall x, F^f(x) = 0 \Rightarrow F^f \in IP

f \notin TOTAL \Rightarrow \exists x, F^f(x)\uparrow \Rightarrow F^f \notin IP

TOTAL \leq_m IP

d.) Show that IP is many-one reducible to TOTAL = \{ f \mid \forall x f(x)\downarrow \}.

Let f be an arbitrary index.

From f, define \forall x, F_f(x) = \exists y [f(x) = 0 \mid f(x) = 1]

f \in IP \Rightarrow \forall x, F_f(x)\downarrow \Rightarrow F_f \in TOTAL

f \notin IP \Rightarrow \exists x, F_f(x)\uparrow \Rightarrow F_f \notin TOTAL