Assignment#4 Key

1. Define PredicateLike(PL) = { f | for each x, f(x) either diverges or evaluates to 0 or 1 }.

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of PL.

$\forall$<x,t. (STP(f, x, t) $\Rightarrow$(VALUE(f, x, t) = 0) | VALUE(f, x, t) = 1))

b.) Use Rice's Theorem to prove that PL is undecidable. Be Complete.

PL is non-trivial as C0(x) = 0 $\in$ PL and S(x) = x+1 $\notin$ PL

Let f,g be two arbitrary indices of procedures such that $\forall$x f(x) = g(x)

f $\in$ PL $\Leftrightarrow$ $\forall$x.(f(x) $\uparrow$ | f(x) = 0| f(x) = 1)

$\Leftrightarrow$ $\forall$x.(g(x) $\uparrow$ | g(x) = 0 | g(x) = 1) since $\forall$xf(x) = g(x

$\Leftrightarrow$ g $\in$ PL

1. Define PredicateLike(PL) = { f | for each x, f(x) either diverges or evaluates to 0 or 1 }.

c.) Show that Diverge = { <f,x> | f(x)↑ } is many-one reducible to PL.

Let f be an arbitrary index and input value.

From <f,x>, define $\forall y\ F_{f,x}(y) = f(x) - f(x) + 2$

$\langle f,x\rangle \in \text{Diverge} \Rightarrow f(x) \uparrow \Rightarrow \forall y\ F_f(y) \uparrow \Rightarrow F_f \in PL$

$\langle f,x\rangle \notin \text{Diverge} \Rightarrow \forall y\ F_f(y) = 2 \Rightarrow F_f \notin PL$

Diverge $\leq_m$ PL

d.) Show that PL is many-one reducible to Diverge = { <f,x> | f(x)↑ }.

Let f be an arbitrary index.

From f, define $\forall x\ F_f(x) = \exists y[f(y) \neq 0\ \&\ f(y) \neq 1]$

$f \in PL \Rightarrow \forall x(\ f(x)\uparrow |\ f(x) = 0 \ |\ f(x) = 1) \Rightarrow \forall x\ F_f(x)\uparrow \Rightarrow F_f \in \text{Diverge}$

$f \notin PL \Rightarrow \exists x(\ f(x)\downarrow\ \&\ f(x) \neq 0\ \&\ f(x) \neq 1) \Rightarrow \exists x\ F_f(x)\downarrow \Rightarrow F_f \notin \text{Diverge}$

PL ≤m Diverge

2. Define IsPredicate(IP) = { f | for each x, f(x) evaluates to 0 or 1 }.

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of IP

$\forall x \exists t$ [STP(f, x,t) & (VALUE(f, x,t) = 0 | VALUE(f, x,t) = 1)]

b.) Use Rice's Theorem to prove that IP is undecidable. Be Complete.

IP is non-trivial as $C_0(x) = 0 \in$ IP and S(x) = x+1 $\notin$IP

Let f, g be two arbitrary indices of procedures such that $\forall x$ f(x) = g(x)。

f $\in$ IP $\Leftrightarrow$ $\forall x$, f(x) = 1 | f(x) = 0

$\Leftrightarrow$ $\forall x$, g(x) = 1 | g(x) = 0 since $\forall x$f(x) = g(x)

$\Leftrightarrow$ g $\in$ IP

2. Define IsPredicate(IP) = { f | $\forall$x, f(x) evaluates to 0 or 1 }.

c.) Show that TOTAL = { f | $\forall$x f(x)$\downarrow$ } is many-one reducible to IP.

Let f be an arbitrary index and input value.

From f, define F_f (x) = f(x) − f(x).

f $\in$ TOTAL $\Rightarrow$ $\forall$x, F_f (x) = 0 $\Rightarrow$ F_f $\in$ IP

f $\notin$ TOTAL $\Rightarrow$ $\exists$x, F_f (x)$\uparrow$ $\Rightarrow$ F_f $\notin$IP

TOTAL $\leq_m$ IP

d.) Show that IP is many-one reducible to TOTAL = { f | $\forall$x f(x)$\downarrow$ } .

Let f be an arbitrary index.

From f, define $\forall$x, F_f(x) = $\exists$y [f(x) = 0 | f(x) = 1)

f $\in$ IP $\Rightarrow$ $\forall$x, F_f(x)$\downarrow$ $\Rightarrow$ F_f $\in$TOTAL

f $\notin$ IP $\Rightarrow$ $\exists$x, F_f(x)$\uparrow$ $\Rightarrow$ F_f $\notin$TOTAL