Assignment#3 Key

1.Consider the language $L = \{a^n b^s c^t \mid s<n \text{ and } t<s\}$. Using the Pumping Lemma for CFLs, show L is not a Context-Free Language.

- Using the Pumping Lemma for CFLs, show L is not a Context-Free Language.
- Assume L is a CFL
- Let N > 0 be from PL
- Chose string $a^{N+2} b^{N+1} c^N$
- PL breaks into uvwxz, $|vwx| \leq N$ and $|vx| > 0$ and says $\forall i \geq 0\ uv^i wx^i z \in L$
- Case 1:. vx contains at least one c. Set i=3, then we at least N+2 c's and only N+2 a's and so string is not in L.
- Case 2: vx contains no c's. Set i = 0, then we still have N c's but one or both of the a's or b's have been reduced and yet N+2 and N+1 are as small as they can be, so the new string is not in L.
- These cases cover all possibilities and so L is not a CFL.

2. Consider some language L. For each of the following cases, write in one of (i) through (vi), to indicate what you can say conclusively about L's complexity, where

(i) L is definitely regular
(ii) L is context-free, possibly not regular, but then again it might be regular
(iii) L is context-free, and definitely not regular
(iv) L might not even be context-free, but then again it might even be regular
(v) L is definitely not regular, and it may or may not be context-free
(vi) L is definitely not even context-free

2b. $L = A \cap B$, where A is context-free, non-regular, and B is regular and of infinite cardinality

L can be characterized by Property (ii), above.

L can be regular: $A = \{ a^n b^n \mid n \geq 0 \}$, $B = a^*$, $L = A \cap B = \{\lambda\}$.

L cab be context-free: $A = \{ a^n b^n \mid n \geq 0 \}$, $B = a^*b^*$, $L = A \cap B = \{ a^n b^n \mid n \geq 0 \}$.

L cannot be a non-CFL as CFLs are closed under intersection with Regular

2a. L = ~(A ⋂B), where A is context-free, non-regular, and B is regular and of infinite cardinality

L can be characterized by Property (iv), above.

L can be regular: A = $\{ a^n b^n \mid n \geq 0 \}$, B = a*, L = ~(A ⋂ B) = {a,b}* - {λ} = $\{a,b\}^+$.

L can be a CFL: A = $\{ a^n b^n \mid n \geq 0 \}$, B = a*b* L = ~(A ⋂ B) = Σ* - $\{ a^n b^n \mid n \geq 0 \}$

= $\{ a^n b^m \mid n \neq m \}$ ∪ { {a,b}* ba {a,b}* } // latter part is regular

L can be non-context-free: A = { wx| w,x ∈ {a,b}*, |w| = |x|, w ≠ x}, B = a*b*,

A ⋂ B = A and ~A = { ww | w ∈ {a,b}* } ∪ {x | x ∈ $\{a,b\}^+$ and has odd length }

2c. A ⊂ L, where A is Regular

L can be characterized by Property (iv), above.

L can be regular: A = Ø *, L = a*.

L can be context-free: A = Ø, L = $\{a^n b^n . | n \geq 0\}$

L can be non-context-free: A = Ø, L = $\{ a^n b^n c^n \mid n \geq 0 \}$.

An even easier explanation is, if A = Ø, A ⊂ L then L can be any set as all sets have Ø as a subset. Even if L is over the simple alphabet {a}, that has an uncountable number of subsets. Thus, L can be absolutely anything, including non-re ones.

3. Show that prfs are closed under sum mutual induction. Sum Mutual Induction means that each induction step, say at y+1, after calculating the base is computed using the sum of the values of the other function up to the y-th value. The formal hypothesis is:

Assume **g1, g2, h1** and **h2** are already known to be prf, then so are **f1** and **f2**, where
f1(x,0) = g1(x); f2(x,0) = g2(x)
f1(x,y+1) = h1(sum(i=0 to y) f2(x, i)); f2(x,y+1) = h2(sum(i=0 to y) f1(x, i))

3. Show that prfs are closed under sum mutual induction. Sum Mutual Induction means that each induction step, say at y+1, after calculating the base is computed using the sum of the values of the other function up to the y-th value. The formal hypothesis is:

$K(x,0) = <g1(x), g1(x), g2(x), g2(x)>$

$K(x,y+1) = \quad <h1(<K(x,y)>_4), +(<K(x,y)>_2, h1(<K(x,y)>_4)),$

$\qquad\qquad\qquad h2(<K(x,y)>_2), +(<K(x,y)>_4, h2(<K(x,y)>_2))>$

$f1(x,y)=< K(x, y) >_1$ // extract first value from quad encoded in K(x,y)

$f2(x,y)=< K(x, y) >_3$ // extract third value from quad encoded in K(x,y)

Why this works

$<K(x,y)>_1$ is f1(x,y), $<K(x,y)>_2$ is sum of f1 values including f1(x,y)

$<K(x,y)>_3$ is f2(x,y), $<K(x,y)>_4$ is sum of f2 values including f2(x,y)