Assign#6 Key

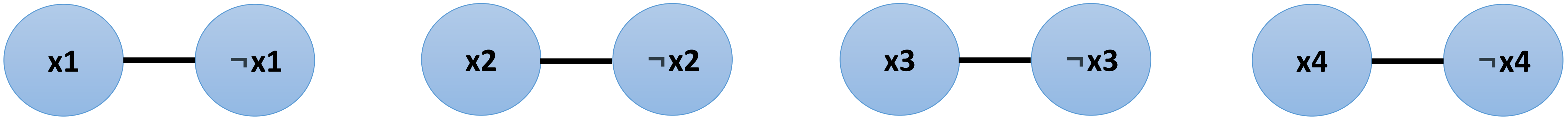
Spring 2022

1. Consider the 3SAT instance:
E = (x1 ∨ x2 ∨ x4) & (¬x1 ∨ ¬x3 ∨ ¬x4) & (¬x2 ∨ ¬x3 ∨ x4) & (¬x2 ∨ ¬x3 ∨ ¬x4)
a. Recast **E** as an instance of k-Vertex Covering and present a solution to the latter
b. Recast **E** as an instance of 3-Coloring and present a solution to the latter

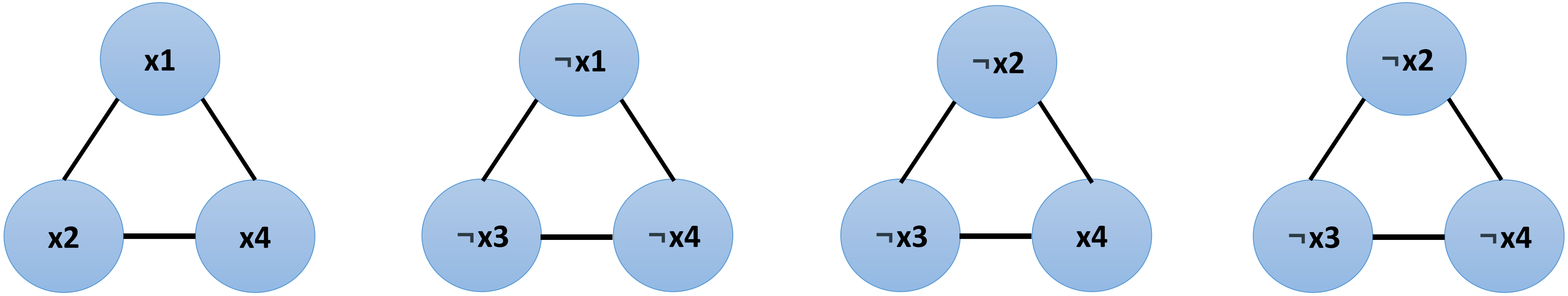
Question 1 (a)

E = (x1 ∨ x2 ∨ x4) & (¬x1 ∨ ¬x3 ∨ ¬x4) & (¬x2 ∨ ¬x3 ∨ x4) & (¬x2 ∨ ¬x3 ∨ ¬x4)

Variable Gadgets:

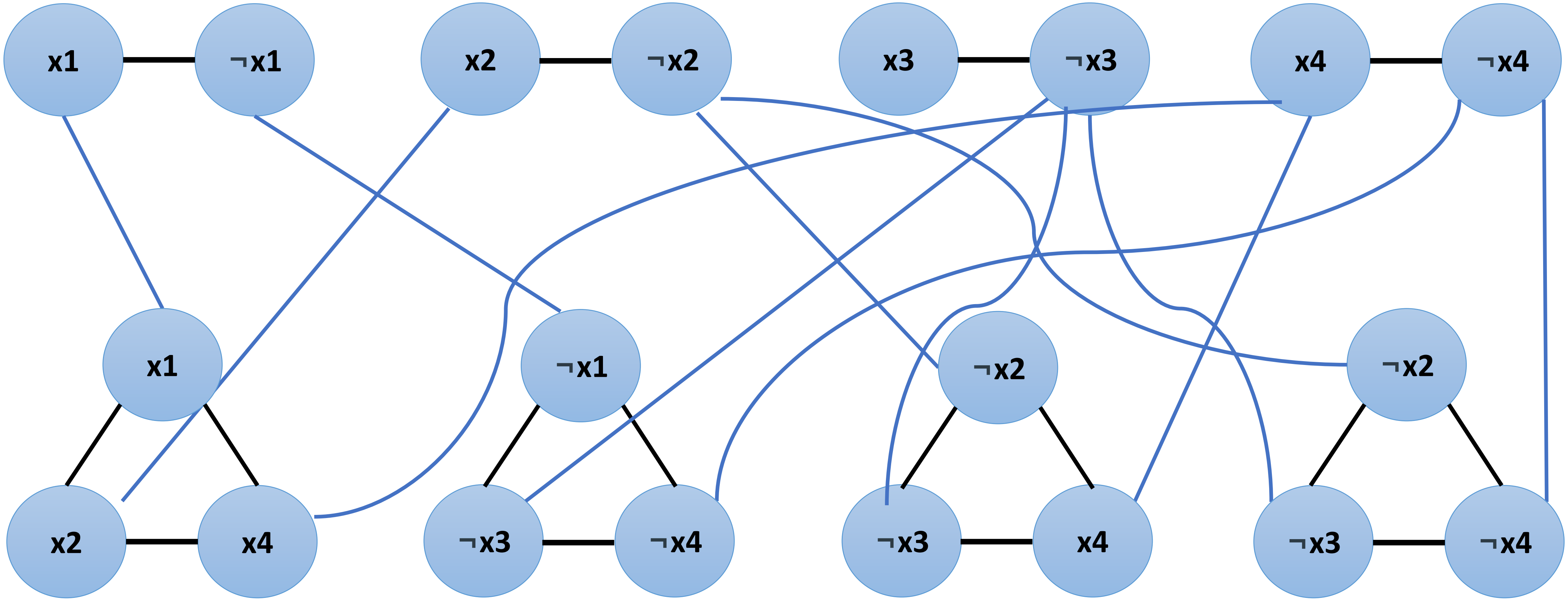


Clause Gadgets:



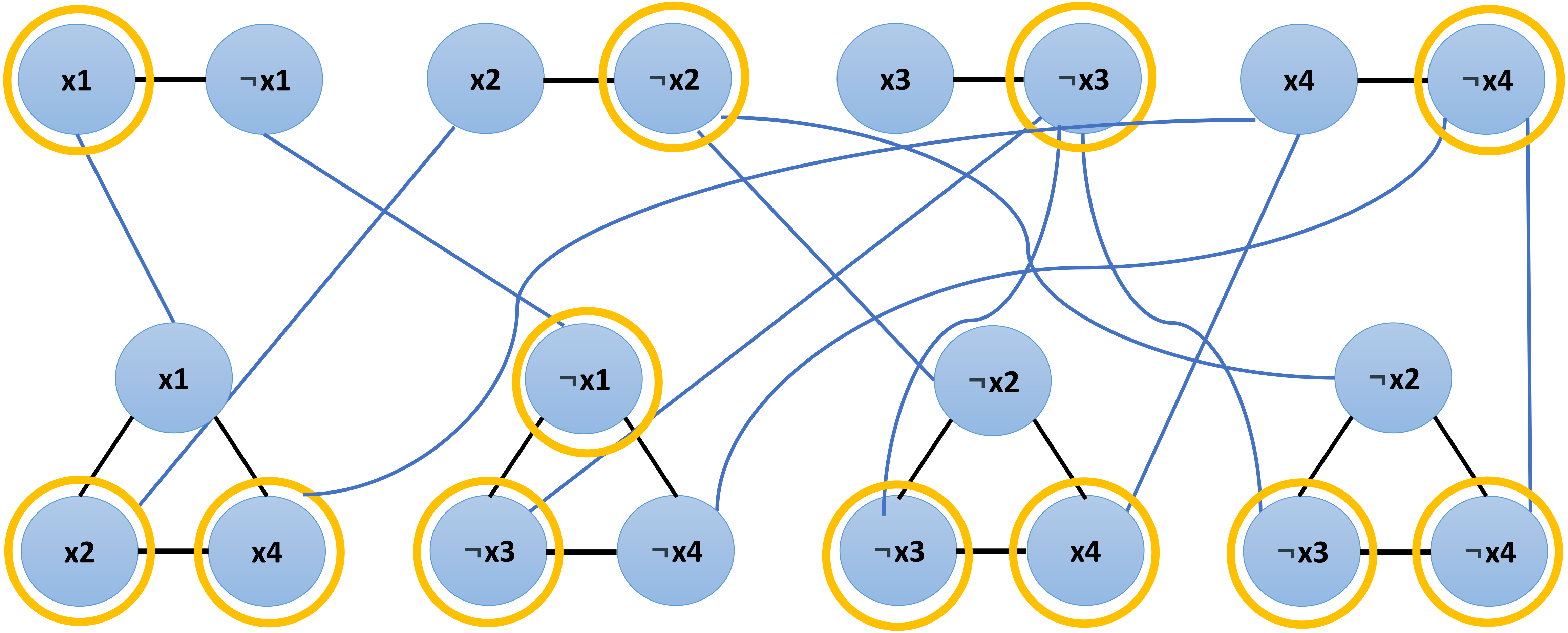
E = (x1 ∨ x2 ∨ x4) & (¬x1 ∨ ¬x3 ∨ ¬x4) & (¬x2 ∨ ¬x3 ∨ x4) & (¬x2 ∨ ¬x3 ∨ ¬x4)

Combined Gadgets:

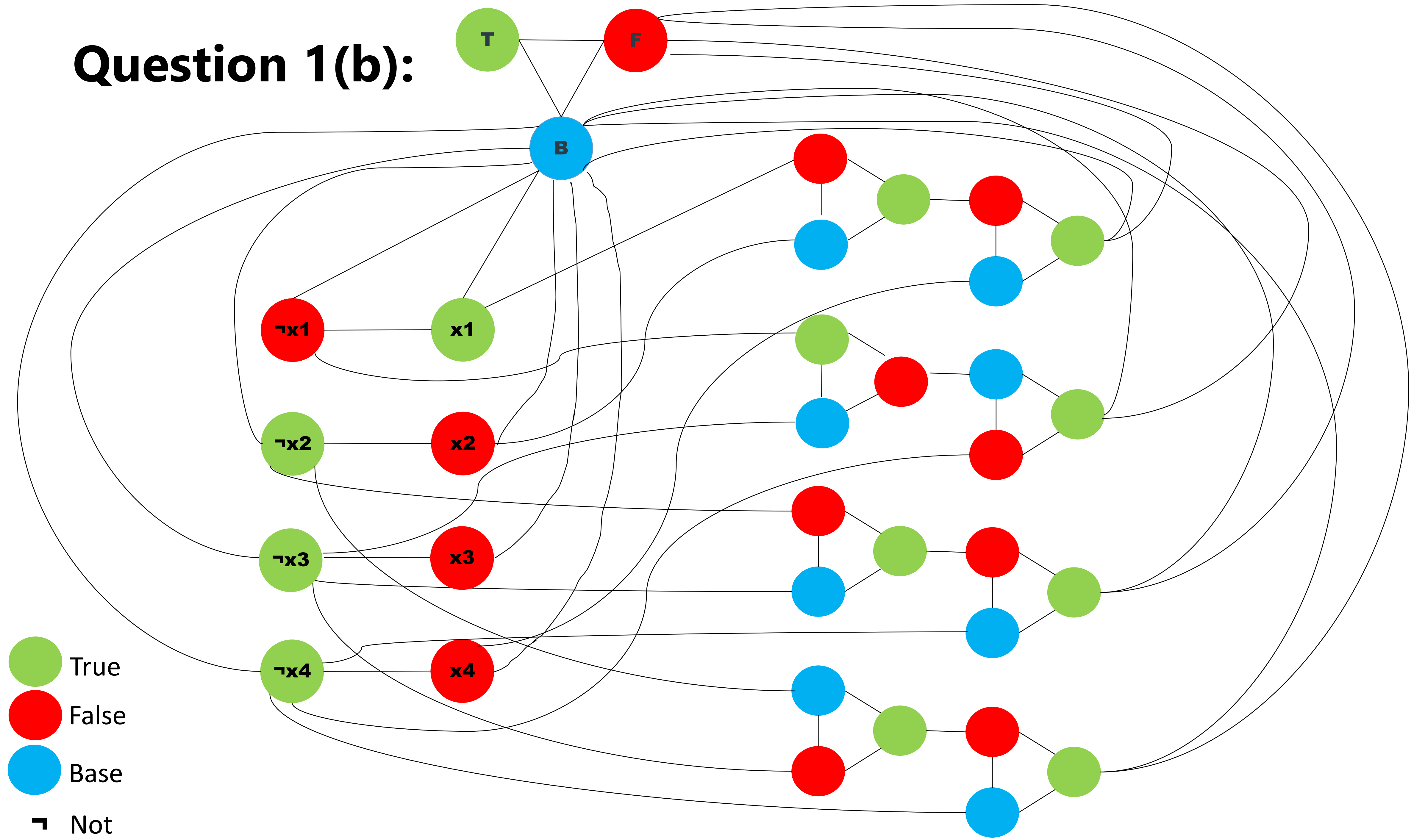


E = (x1 ∨ x2 ∨ x4) & (¬x1 ∨ ¬x3 ∨ ¬x4) & (¬x2 ∨ ¬x3 ∨ x4) & (¬x2 ∨ ¬x3 ∨ ¬x4)

Selecting Vertex Cover:



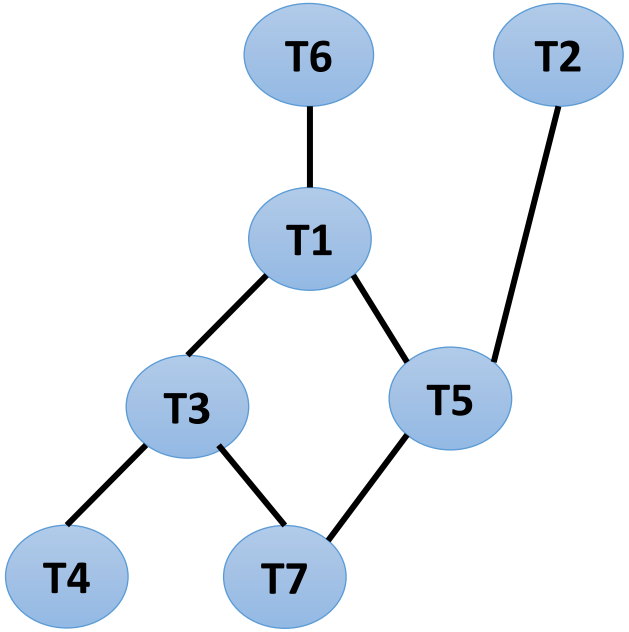
Question 1(b):



2. Task set **(T1,2), (T2,1), (T3,1), (T4,3), (T5,3), (T6,2), (T7,5)**, with partial order

T1<T3; T1<T5, T2<T5, T3<T4; T3<T7; T6<T1; T5<T7

a. Draw the graph that depicts these relationships.



b. Show the 2-processor schedule that results when the task number is the priority; a smaller task number means higher priority.

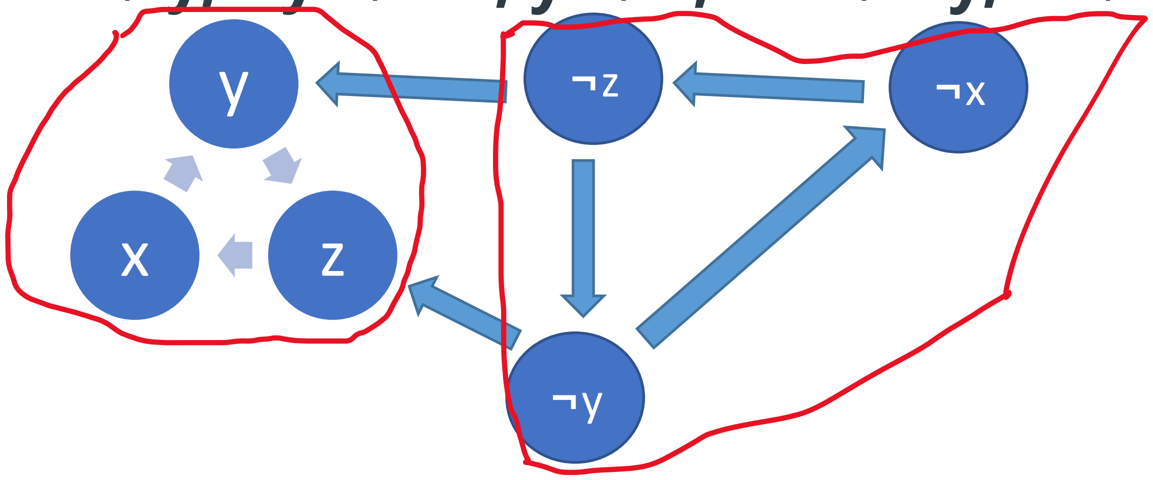
T2		T1	T1	T3	T4	T4	T4												
T6	T6			T5	T5	T5	T7	T7	T7	T7	T7								

3. Consider the following 2SAT instance.

(¬x ∨ y) (¬y ∨ z) (¬z ∨ x) (z ∨ y)

a. Draw the implication graph associated with this formula.

x → y; ¬y → ¬x; y → z; ¬z → ¬y; z → x; ¬x → ¬z; ¬z → y; ¬y → z

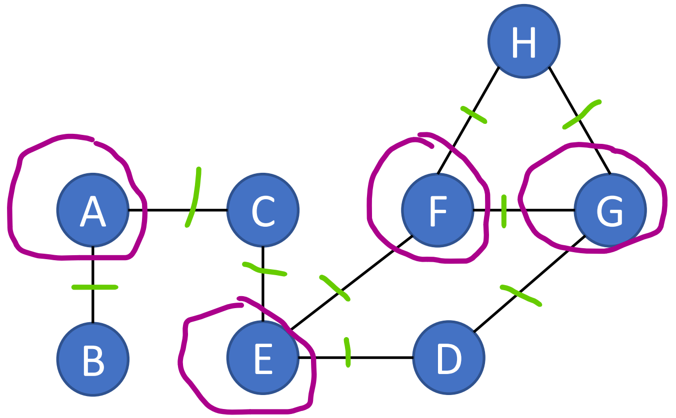


b. Draw circles around the strongly connected components (see red circles)

c. Provide a solution based on the SCCs or highlight the conflict exposed by the SCCs – the cluster with three elements has no outgoing edges, so

x = y = z = T

4. Consider the following instance of Positive Min-Ones-2SATt,
(A ∨ B) (A ∨ C) (C ∨ E) (D ∨ E) (D ∨ G) (E ∨ F) (F ∨ G) (F ∨ H) (G ∨ H)
a. Convert this instance of Positive 2SAT to a graph for which Min Vertex Cover is equivalent to the Min-Ones problem.



b. Show solution for Min Vertex Cover for (a) and correspondingly for the Positive Min-Ones-2SAT instance.

Solution: Min Cover is 4 choosing **A, E, F, G**; True assignments are is **A = E = F = G = T**

See circled nodes and covered edges with green slashes.