Consider the SAT instance:

(x1 ∨ x2 ∨ x4 ∨ x5) & (¬x1 ∨ ¬x2 ∨ x3 ∨ ¬x4 ∨ ¬x5) & (x1 ∨ ¬x4)

1. Recast this as an instance of 3SAT.

ANS:

(x1 ∨ x2 ∨ x6) & (x4 ∨ x5 ∨ ¬x6) & (¬x1 ∨ ¬x2 ∨ x7) & (x3 ∨ ¬x4 ∨ x8) & (¬x5 ∨ ¬x7 ∨ ¬x8) & (x1 ∨ ¬x4 ∨ x1)

ANS:

c1 = (x1 ∨ x2 ∨ x6)
c2 = (x4 ∨ x5 ∨ ¬x6)
c3 = (¬x1 ∨ ¬x2 ∨ x7)
c4 = (x3 ∨ ¬x4 ∨ x8)
c5 = (¬x5 ∨ ¬x7 ∨ ¬x8)
c6 = (x1 ∨ ¬x4 ∨ x1)

A simple solution is **x1, x2, x3, x4, x5, x6, x7, ¬x8**

2. Construct the SubsetSum instance equivalent to this and state what rows must be chosen.

(x1 ∨ x2 ∨ x6) & (x4 ∨ x5 ∨ ¬x6) & (¬x1 ∨ ¬x2 ∨ x7) & (x3 ∨ ¬x4 ∨ x8) & (¬x5 ∨ ¬x7 ∨ ¬x8) & (x1 ∨ ¬x4 ∨ x1)

	x1	x2	x3	x4	x5	x6	x7	x8	C1	C2	C3	C4	C5	C6
x1	1	0	0	0	0	0	0	0	1	0	0	0	0	2
~x1	1	0	0	0	0	0	0	0	0	0	1	0	0	0
x2	0	1	0	0	0	0	0	0	1	0	0	0	0	0
~x2	0	1	0	0	0	0	0	0	0	0	1	0	0	0
x3	0	0	1	0	0	0	0	0	0	0	0	1	0	0
~x3	0	0	1	0	0	0	0	0	0	0	0	0	0	0
x4	0	0	0	1	0	0	0	0	0	1	0	0	0	0
~x4	0	0	0	1	0	0	0	0	0	0	0	1	0	1
x5	0	0	0	0	1	0	0	0	0	1	0	0	0	0
~x5	0	0	0	0	1	0	0	0	0	0	0	0	1	0
x6	0	0	0	0	0	1	0	0	1	0	0	0	0	0
~x6	0	0	0	0	0	1	0	0	0	1	0	0	0	0
x7	0	0	0	0	0	0	1	0	0	0	1	0	0	0
~x7	0	0	0	0	0	0	1	0	0	0	0	0	1	0
x8	0	0	0	0	0	0	0	1	0	0	0	1	0	0
~x8	0	0	0	0	0	0	0	1	0	0	0	0	1	0
C1	0	0	0	0	0	0	0	0	1	0	0	0	0	0
C1’	0	0	0	0	0	0	0	0	1	0	0	0	0	0
C2	0	0	0	0	0	0	0	0	0	1	0	0	0	0
C2’	0	0	0	0	0	0	0	0	0	1	0	0	0	0
C3	0	0	0	0	0	0	0	0	0	0	1	0	0	0
C3 ‘	0	0	0	0	0	0	0	0	0	0	1	0	0	0
C4	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C4’	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C5	0	0	0	0	0	0	0	0	0	0	0	0	1	0
C5’	0	0	0	0	0	0	0	0	0	0	0	0	1	0
C6	0	0	0	0	0	0	0	0	0	0	0	0	0	1
C6’	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	3	3	3	3	3	3

3. Recast the SubsetSum instance in Part 2 as a Partition instance (really easy). Show the Partitioning into equal subsets.

Ans:
G = 11111111333333
sum= 22222222555555
$2 * \text{sum} - G =$ 33333333777777
sum + G = 33333333888888
sum is the sum of all rows.
Note: If you use 1 in X1/C6 then
sum is 22222222555554 and so
$2 * \text{sum} - G =$ 33333333777775
sum + G = 33333333888887
The partitions for the case where we use 2 in x1/C6 are as follows:

Partition 1:

33333333 777777	2*sum -G
10000000 100002	x1
01000000 100000	x2
00100000 000100	x3
00010000 010000	x4
00001000 010000	x5
00000100 100000	x6
00000010 001000	x7
00000001 000010	¬x8
00000000 010000	C2
00000000 010000	C3
00000000 001000	C3’
00000000 000100	C4
00000000 000010	C5
00000000 000010	C5’
00000000 000001	C6

c1 = (x1 V x2 V x6)
c2 = (x4 V x5 V ¬x6)
c3 = (¬x1 V ¬x2 V x7)
c4 = (x3 V ¬x4 V x8)
c5 = (¬x5 V ¬x7 V ¬x8)
c6 = (x1 V ¬x4 V x1)

A simple solution is **x1, x2, x3, x4, x5, x6, x7, ¬x8**

Partition 2:

33333333 888888	sum+G
10000000 001000	~x1
01000000 001000	~x2
00100000 000000	~x3
00010000 000101	~x4
00001000 000010	~x5
00000100 010000	~x6
00000010 000010	~x7
00000001 000100	x8
00000000 100000	C1
00000000 100000	C1’
00000000 010000	C2’
00000000 000100	C4’
00000000 000001	C6’

c1 = (x1 V x2 V x6)
c2 = (x4 V x5 V ¬x6)
c3 = (¬x1 V ¬x2 V x7)
c4 = (x3 V ¬x4 V x8)
c5 = (¬x5 V ¬x7 V ¬x8)
c6 = (x1 V ¬x4 V x1)

A simple solution is **x1, x2, x3, x4, x5, x6, x7, ¬x8**

4. Recast the original SAT as a 0-1 Integer Linear Programming instance:

$(x1 \lor x2 \lor x4 \lor x5) \,\&\, (\neg x1 \lor \neg x2 \lor x3 \lor \neg x4 \lor \neg x5) \,\&\, (x1 \lor \neg x4)$

ANS:

Assume 0 <= x1, x2, x3, x4, x5 <= 1

x1 + x2 + x4 + x5 >= 1

(1-x1) + (1-x2) + x3 + (1-x4) + (1-x5) >= 1

x1 + (1-x4) >= 1

We choose: **x1 = 1, x2 = 1, x3 = 1, x4 = 1, x5 = 1**

5. Consider the following set of independent tasks with associated task times:

(T1,3), (T2,5), (T3,7), (T4,6), (T5,2), (T6,8), (T7,1)

Fill in the schedules for these tasks under the associated strategies below.

Greedy using the list order above:

Greedy using a reordering of the list so that longest-running tasks appear earliest in the list:

Greedy then sorted high to low

T1	**T1**	**T1**	**T3**	**T3**	**T3**	**T3**	**T3**	**T3**	**T3**	**T5**	**T5**	**T7**							
T2	T2	T2	T2	T2	T4	T4	T4	T4	T4	T4	T6	T6	T6	T6	T6	T6	T6	T6	

(T1,3), (T2,5), (T3,7), (T4,6), (T5,2), (T6,8), (T7,1)

T6	**T6**	**T6**	**T6**	**T6**	**T6**	**T6**	**T6**	**T2**	**T2**	**T2**	**T2**	**T2**	**T1**	**T1**	**T1**				
T3	T3	T3	T3	T3	T3	T3	T4	T4	T4	T4	T4	T4	T5	T5	T7				

(T6,8), (T3,7), (T4,6), (T2,5), (T1,3), (T5,2), (T7,1)