Assignment#4 Key

1. HAS_SUC(HS) = { f | ∃x, f(x)↓, and f(x) = x+1 }

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **HS.**

∃<x,t> [STP(f,x,t) & (VALUE(f,x,t) = x+1)]}

b.) Use Rice's Theorem to prove thatHSis undecidable. Be Complete.

HS is non-trivial as **S(x) = x+1** ∈ **ND** and **C0(x) = 0** ∉ **ND**

Let f,g be two arbitrary indices of procedures such that ∀**x f(x) = g(x)**

f ∈ **HS** iff ∃**x f(x) = x+1 iff** $f(x_0) = x_0 + 1$ **for some** x_0 **iff** $g(x_0) = x_0 + 1$ **as** ∀**x f(x) = g(x) implies** ∃**x g(x) = x + 1 iff g** ∈ **HS**

f ∉ **HS** iff ∀**x [f(x)↓ implies f(x) ≠ x + 1] iff** ∀**x [g(x)↓ implies g(x ≠ x + 1] as** ∀**x f(x) = g(x) iff g** ∉ **ND**

c.) Show that **K = { f | f(f)** converges **}** is many-one reducible to **HS**.

Let **f** be an arbitrary index. From **f**, define ∀**x** F_f**(x) = f(f)-f(f) + x + 1**.
f ∈ **K** implies ∀**x** F_f**(x) = x + 1** implies F_f ∈ **HS**.
f ∉ **K** implies ∀**x** F_f**(x)** diverges implies F_f ∉ **HS**.

Thus, $K \leq_m HS$

d.) Show that **HS** is many-one reducible to **K = { f | f(f)** converges **}**

Let **f** be an arbitrary index. From **f**, define ∀**y** F_f**(y) =** ∃**<x,t> STP(f,x,t) & (VALUE(f,x,t) = x + 1)**
f ∈ **HS** implies ∀**y** F_f**(y)** converges implies $F_f(F_f)$ converges implies F_f ∈ **K**
f ∉ **HS** implies ∀**y** F_f**(y)** diverges implies F_f ∉ **K**.

Thus, $HS \leq_m K$

2. IS_SUC(IS) = { f | ∀x, f(x)↓, and f(x) = x+1 }

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **IS**.

∀x ∃t [STP(f,x,t) & (VALUE(f,x,t) = x + 1)]}

b.) Use Rice's Theorem to prove that **IS** is undecidable. Be Complete.

AD is non-trivial as **S(x) = x+1** ∈ **AD** and **C0(x) = 0** ∉ **AD**

Let f,g be two arbitrary indices of procedures such that ∀**x f(x) = g(x)**

f ∈ **IS** iff ∀**x f(x) = x + 1 iff** ∀**x g(x) = x + 1 iff g** ∈ **IS**

c.) Show that **TOT = { f |** for all **x**, f**(x)** converges **}** is many-one reducible to **IS**.

Let **f** be an arbitrary index. From **f**, define ∀**x** F_f**(x) = f(x)-f(x) + x + 1**.
f ∈ **TOT** implies ∀**x** F_f**(x) = x+1** implies F_f ∈ **HS**.
f ∉ **TOT** implies ∃**x** F_f**(x)** diverges implies F_f ∉ **HS**.

Thus, **TOT** $\leq_m$ **HS**

d.) Show that **HS** is many-one reducible to **TOT = { f |** for all **x**, f**(x)** converges **}**

Let **f** be an arbitrary index. From **f**, define ∀**x** F_f**(x) = μy [f(x) = x + 1]**
f ∈ **IS** implies ∀**x** F_f**(x)** converges implies F_f ∈ **TOT**
f ∉ **IS** implies ∃**x** F_f**(x)** diverges implies F_f ∉ **TOT**.

Thus, **IS** $\leq_m$ **TOT**