Sample Assignment#2 Key
1a. **EveryOther**(L) = \{ a_1 \ a_3 \ ... \ a_{2n-1} \ | \\
a_1 \ a_2 \ a_3 \ ... \ a_{2n-1} \ a_{2n} \text{ is in } L \} \\

- Approach 1: Let L be a Regular language over the finite alphabet \( \Sigma \). For each \( a \in \Sigma \), define \( f(a) = \{ a, a' \} \), \( g(a) = a' \) and \( h(a) = a \), \( h(a') = \lambda \). \( f \) is a substitution, \( g \) and \( h \) are homomorphisms.

\[
\text{EveryOther}(L) = h(f(L) \cap (\Sigma \cdot g(\Sigma))^*)
\]

- Why this works:
  \( f(L) \) gets us every possible random priming of letters of strings in L.
  \((\Sigma \cdot g(\Sigma))^*\) gets every word composed of pairs of unprimed and primed letters from \( \Sigma \). Intersecting this with \( f(L) \) gets strings of the form \( a_1 \ a_2' \ a_3 \ a_4' \ldots a_{2n-1} a_{2n}' \) where \( a_1 \ a_2 \ a_3 \ a_4 \ldots a_{2n-1} a_{2n} \) is in L.
  Applying the homomorphism \( h \) erases all primed letters resulting in every string \( a_1 \ a_3 \ldots a_{2n-1} \) where \( a_1 \ a_2 \ a_3 \ a_4 \ldots a_{2n-1} \ a_{2n} \) is in L, precisely the language \( \text{EveryOther}(L) \) that we sought. This works as Regular Languages are closed under intersection, concatenation, *, substitution and homomorphism.
1a. EveryOther(L) = \{ a_1 a_3 \ldots a_{2n-1} \mid a_1 a_2 a_3 \ldots a_{2n-1} a_{2n} \text{ is in } L \}

• Approach 2: Let L be a Regular language over the finite alphabet \( \Sigma \). Assume L is recognized by the DFA \( A_1 = (Q, \Sigma, \delta_1, q_1, F) \). Define NFA \( A_2 = (Q, \Sigma, \delta_2, q_1, F) \), where \( \delta_2(q,a) = \text{union}(b \in \Sigma) \{ \delta_1(\delta_1(q,a),b) \} \)

• Why this works:
  Every transition that \( A_2 \) takes is one that \( A_1 \) would have taken when reading a pair that starts with the character read by \( A_1 \) followed by any arbitrary character.
1b. \( \text{Half}(L) = \{ x \mid \text{there exists a } y, \ |x| = |y| \text{ and } xy \text{ is in } L \} \)

- Let \( L \) be a Regular language over the finite alphabet \( \Sigma \). Assume \( L \) is recognized by the DFA \( A_1 = (Q, \Sigma, \delta_1, q_1, F) \). Define the NFA \( A_2 = ((Q \times Q \times Q) \cup \{ q_0 \}, \Sigma, \delta_2, q_0, F') \), where
  \[
  \delta_2(q_0, \lambda) = \text{union}(q \in Q) \{<q_1, q, q>\} \text{ and }
  \delta_2(<q, r, s>, a) = \text{union}(b \in \Sigma) \{<\delta_1(q, a), \delta_1(r, b), s>\}, \ q, r, s \in Q
  \]
  \[F' = \text{union}(q \in Q) \{<q, f, q>\}, f \in F\]

- Why this works:
The first part of a state \(<q, r, s>\) tracks \( A_1 \).
The second part of a state \(<q, r, s>\) tracks \( A_1 \) for precisely all possible strings that are the same length as what \( A_1 \) is reading in parallel. This component starts with a guess as to what state \( A_1 \) might end up in.
The third part of a state \(<q, r, s>\) remembers the initial guess.
Thus, \( \delta_2(<q_1, q, q>, x) = \{\delta_1(q_0, x), \delta_1(q, y), q>\} \) for arbitrary \( y, |x| = |y| \)
We accept if the initial guess was right and the second component is final, meaning \( xy \) is in \( L \).
2. \( L = \{ a^m b^n c^t \mid t = \min(m,n) \} \)

a.) Use the **Myhill-Nerode Theorem** to show \( L \) is not Regular.

Define the equivalence classes \([a^i b^i], i \geq 0\)

Clearly \( a^i b^i c^i \) is in \( L \), but \( a^j b^j c^i \) is not in \( L \) when \( j \neq i \)

Thus, \([a^i b^i] \neq [a^j b^i]\) when \( j \neq i \) and so the index of \( R_L \) is infinite.

By Myhill-Nerode, \( L \) is not Regular.
2. \[ L = \{ a^m b^n c^t \mid t = \min(m,n) \} \]

b.) Use the **Pumping Lemma for CFLs** to show \( L \) **is not** a CFL

Me: \( L \) is a CFL

PL: Provides \( N > 0 \)

Me: \( z = a^N b^N c^N \)

PL: \( z = uvwxy, \ |vwx| \leq N, \ |vx| > 0, \) and \( \forall i \geq 0 \ uv^iwx^iy \in L \)

Me: Since \( |vwx| \leq N \), it can consist of \( a \)'s and/or \( b \)'s or \( b \)'s and/or \( c \)'s but never all three.

Assume it contains no \( c \)'s then \( i = 0 \) decreases the number of \( a \)'s and/or the number of \( b \)'s, but not the \( c \)'s and so there are more \( c \)'s than the minimum of \( a \)'s and \( b \)'s.

Assume it contains \( c \)'s then \( i = 2 \) increases the number of \( c \)'s and maybe number of \( b \)'s, but not the \( a \)'s and so there are more than \( N \) \( c \)'s but just \( N \) \( a \)'s.
2. \( L = \{ a^m b^n c^t \mid t = \min(m,n) \} \)

c.) Present a CSG for \( L \) to show it is context sensitive

\[
G = ( \{ A, B, C, <a>, <b> \}, \{ a, b, c \}, R, A )
\]

\[
A \rightarrow aBbc | abc | a | b | \lambda
\]

\[
B \rightarrow aBbC | abC | a<a>bC | ab<b>C
\]

\[
C_b \rightarrow bC \quad \text{// Shuttle C over to a c}
\]

\[
C_c \rightarrow cc \quad \text{// Change C to a c}
\]

\[
<a> \rightarrow a<a> | a
\]

\[
<b> \rightarrow b<b> | b
\]