Midterm Exam

(!) This is a preview of the published version of the quiz

Started: Apr 19 at 11:20pm

Quiz Instructions

This is the Midterm exam and it is 100% on-line. As I indicated previously, you are not to discuss this exam with anyone until after the exam time is completed and, even then, only with students who also took the exam until grades are posted. You may use no resources that are not from this course. That means you can use your personal notes, all the material linked from the course website, and anything at this Webcourses site. You cannot go hunting on the Internet for answers. You cannot communicate with anyone while taking the exam. You cannot share the contents of this exam. You cannot make copies of the exam using screen capture or any other method. You cannot post questions or answers from the exam on any forum. Before taking the exam you must agree to all the conditions set in the Honor Code quiz associated with this course.

Note that Canvas loves to number all questions sequentially so my 11.1, 11.2, 12.1, 12.2, and 12.3 get numbered 11-15. That's annoying but not a big deal and I keep consistent numbering in the questions themselves.

All the best!!

Question 1		10 pts
1. This is a collection of 10 True/False Que	estions.	
Membership in Phrase-Structured Languages is semi-decidable	TRUE	•
Membership in Context-Free Languages can be solved in polynomial time	TRUE	•
Every recursive (decidable) set is recognized by some primitive recursive function	FALSE	 Image: A start of the start of
Every non-empty re set is enumerated by some primitive recursive function	TRUE	 Image: A start of the start of
An algorithm exists to determine if a Context- Sensitive Language is finite	FALSE	•
The Context-Free Languages are closed under intersection	FALSE	•
We can decide if a Context-Free Grammar generates the empty set	TRUE	•
If S is a decidable set and R is a subset of S, then R is also decidable	FALSE	•

The quotient of a Context-Free Language and a Regular Language is Context-Free	TRUE	~
Deterministic PDAs have the same power as Non- Deterministic PDAs	FALSE	~

Question 2	8 pts
The general form of such expressions i quantifiers. Usually, the predicate is Pri a set (membership in it is the problem)	sed to determine the upper bound of the complexity of some problem. s Q [Algorithmic Predicate } . Here Q is a sequence of alternating mitive Recursive involving the functions STP and VALUE . I will give you and an associated predicate. The Quantifier part won't be specified, and otions, one of which specifies what that quantifier is.
S = { f domain(f) is non-empty }.	
$f \in S \text{ iff} \exists \langle x, t \rangle$	[STP(f,x,t)]
Let T = { <f,x> 2x is in range(f) }. <f,x> ∈ T iff ∃ <y,t></y,t></f,x></f,x>	✓ [STP(f,y,t) & VALUE(f,y,t)=2x]
Let U = { f Range(f) contains no odd	I numbers }.
$f \in U \text{ iff } \forall $	$[STP(f,x,t) \Rightarrow EVEN(VALUE(f,x,t))]$
Let V= { f Domain(f) is not the set of	
$\mathbf{f} \in \mathbf{V}$ iff $\exists x \forall t$	[~STP(f,x,t)]

Question 3

5 pts

3. Infinite Recursively Enumerable Sets

Let set **S** be a **recursive enumerable**, possibly **recursive (decidable)**, **non-empty** set. Further, let **S** be the range of some algorithm f_S . Define f_R iteratively as follows:

$$\begin{split} f_{\rm R}(0) &= f_{\rm S}(0) \\ f_{\rm R}(y{+}1) &= f_{\rm S}(\ \mu \ z \ [\ f_{\rm S}(z) > f_{\rm R}(y) \] \end{split}$$

Clearly, f_R enumerates a set R that is in some way related to S. That relation and various properties of R and S are the topics of the following question where you must select all true properties and not select the false ones.

✓ If S is infinite, f _R is a monotonically increasing function
✓ S is finite if and only if R is finite
□ f _R halts for all input (is an algorithm)
✓ R is a decidable (recursive) set
□ f _R is a primitive recursive function

Question 4

5 pts

4. Rice's Theorem (Variations)

Match the problems about sets of function indices with the statements about the applicability of Rice's Theorems (including its three variants and circumstances under which no version is applicable). Note, your answer must always choose a Weak Form if one is applicable.

{ f for every x, $f(x) < x$ }	Strong Version of Rice's Thec V	
{ f for every x, f(2x) converges and f(2x+1) diverges }	Domain (Weak) Version of Ric 🗸	
{ f for some x, f(x) converges in at most x steps }	Rice's Theorem does not app v	
{ f for every x, there is some y such that $f(y) = x$ }	Range (Weak) Version of Rice 🗸	
{ f there is some g not equal to f where, for some x, $f(x) = g(x)$ or both diverge }	Rice's Theorem does not app 🗸	

Question 5		5 p	ots
v	blved , solvable but unsolved, l RE (not semi-decidable or eve	RE (semi-decidable) but not solvable, Co-RE be en complement of semi-decidable). Specify the	ut
Is P = NP?	Solvable	~	

What is the minimum number of states for some DFA?	Is f(x) = 2x, for some arbitrary procedure f and input x?	RE	~
DFA?	Is f an algorithm, for arbitrary procedure f?	Non-RE/Non-Co-RE	~
Is domain(f) empty, for arbitrary procedure f?	What is the minimum number of states for some DFA?	Solved	~
	Is domain(f) empty, for arbitrary procedure f?	Co-RE	~

Question 6	5 pts
6. Regular Expressions	
The unique solution to the regular equation $A = 0 + A1$ is 01^* \checkmark .	
The unique solution to the regular equation $A = 0 + A1^*$ is Non-existent	
The k in $R_{i,i}^{k}$ is the Max index of any intermediat \checkmark .	
·,j	
The i in $R_{i,i}^{k}$ is the Starting State \checkmark .	
·j	
The values i and j in R _{i,i} k can v be equal	

Question 7	4 pts
7. Pumping Lemmas There is a Pumping Lemma for Regular Languages and a separate one for Context-Free The Pumping Lemma for Context-Free Languages degenerates to the Pumping Lemma	
when The alphabet has just one lett v.	
The Pumping Lemma for Context-Free Languages assumes the grammar is in Chomsky	y Normal 🗸
Myhill-Nerode provides an alternative to the Pumping Lemma for Regular	✓ Languages

Question 8

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Both Pumping Lemmas depend on the	Pigeon Hole 🗸	Principle

8. Meta-Theorem on Closures of Regular and Context-Free Languages

What language is gotten from each of these uses of the meta-theorem we developed from knowing the Regular and Context-Free Languages are both closed under Substitution, Homomorphism, and Intersection with Regular Languages? In each case, for $\mathbf{a} \in \Sigma$, we define $\mathbf{g}(\mathbf{a}) = \mathbf{a}'$, $\mathbf{f}(\mathbf{a}) = \{\mathbf{a}, \mathbf{a}'\}$, and $\mathbf{h}(\mathbf{a}) = \mathbf{a}$, $\mathbf{h}(\mathbf{a}') = \lambda$. Here \mathbf{a}' is a new symbol uniquely associated with \mathbf{a} but not appearing in Σ . We then extend each of these in a natural way to apply to strings and do closure constructions for some operation using the form:

h(f(L) \cap E), where E is some Regular Expression and L is some non-empty Context-Free Language

What is the language we get when E is $\Sigma^* \mathbf{g}(\Sigma^*)$? Prefix(L)	
What is the language we get when E is $g(\Sigma^*)$? String of length 0	
What is the language we get when E is $g(\Sigma^*)$ $\Sigma^* g(\Sigma^*)$? Substring(L)	~
What is the language we get when E is $g(\Sigma^*)$ Σ^* ? Suffix(L)	
What is the language we get when E is Σ^* ?	

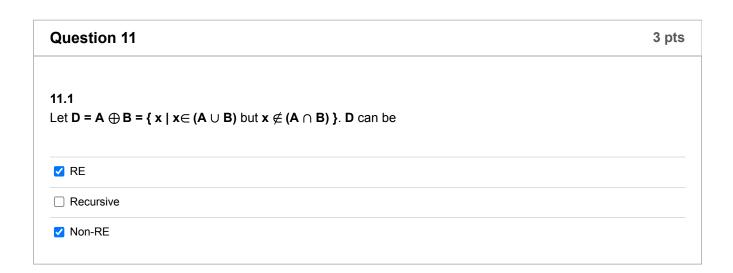
Question 9	5 pts
9.	
PCP was used to show the undecidability of which of the following?	
Is the language derived by some CSG non-empty?	
Does some CFG produce all strings over its alphabet?	
✓ Is the intersection of two CFLs non-empty?	
✓ Is an arbitrary CFG ambiguous?	
□ Is the intersection of two CFLs a non-CFL?	

5 pts

Question 10		3 pts
10.		
The traces of terminal computations prod	luced by running Turing Machines	are Context-Sensitive 🗸
but the complements of these traces are	Context-Free ~	The reason is that the easier case is a
there exists ~ condition		

11. Closure Properties of Sets over the Natural Numbers.

Let set **A** be a **non-empty recursive** (**decidable**) set, let **B** be an **RE non-recursive** set, and let **C** be a **non-RE** set. Specify, for each set below, which can be **recursive**, **RE**, and/or **non-RE**. By can be, I mean there are some choices of the sets **A**, **B**, and **C**, used to construct the new set, that can make it **recursive**, **RE**, and/or **non-RE**. For each of the following two questions, specify what are the possible results of the given operation.



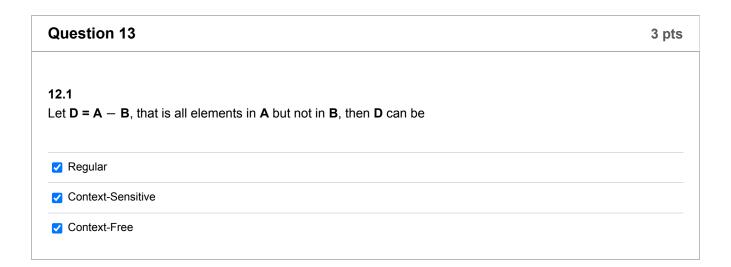
Question 12	3 pts
11.2 Let $E = \{ z \mid z = y / x where x \in A, y \in B \}$. Rec value divided by 0 is 0. E can be	all that // is just integer division with the odd exception that any
Z RE	

□ Non-RE

Recursive

12. Closure Properties of Languages over some alphabet $\boldsymbol{\Sigma}$.

Consider **non-empty** languages (sets) **A**, **B**, and **C**, where **A** is **Regular**, **B** is **Context-Free**, and **C** is **Context-Sensitive**. Specify, for each set below, which can be **Regular**, **Context-Free**, and/or **Context Sensitive**. By can be, I mean there are some choices of the sets **A**, **B**, and **C** used to construct the new set that can make it **Regular**, **Context-Free**, and/or **Context-Sensitive**. For each of the following four questions, specify what are the possible results (more than one might be true) of the given operation.



Question 14	3 pts
12.2 Let E = A \cap B , that is all elements in both A and B , then E can be	
Context Sensitive	
Z Regular	

Question 15	3 pts

2.3 et F = Σ* − C , that	is all elements no	ot in C can be		
Context Free				
Regular				
Context Sensitive				

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