Key for Assign#4
HasExp(HE)={ f | \exists x \ f(x) \downarrow \ & \ f(x)=2^x \ }

Show a minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of HE.

\[ \exists<x,t> \ [ \text{STP}(f,x,t) \ & \ (\text{VALUE}(f,x,t) = 2^x) \ ] \]

Thus, \( \text{HD} \leq_m \text{K}_0 \)
Use Rice’s Theorem to prove that HD is undecidable. Be Complete.

HD is non-trivial as C1(x) = 1 ∈ HE as C1(0) = 1 = 2^0
and C0(x) ∉ HE

Let f, g be two arbitrary indices of procedures such that
∀x f(x) = g(x)

f ∈ HE ⇔ ∃x [ f(x)↓ & f(x) = 2^x ]
⇔ ∃x [ g(x)↓ & g(x) = 2^x ]
   as ∀x f(x) = g(x) and so g has same I/O properties
⇔ g ∈ HE

f ∉ HE ⇔ ∀x [ f(x)↓ ⇒ f(x) ≠ 2^x ]
⇔ ∀x [ g(x)↓ ⇒ g(x) ≠ 2^x ]
   as ∀x f(x) = g(x) and so g has same I/O properties
⇔ g ∉ HE
Show that $\text{HAS} \_\text{ID} (\text{HI}) = \{ f \mid \exists x \ f(x) \downarrow \& f(x) = x \}$ is many-one reducible to $\text{HE}$.

Let $f$ be an arbitrary index

From $f$, define $\forall x \ F_f(x) = | f(x) - x | + 2^x$

$f \in \text{HI} \Rightarrow \exists x \ F_f(x) = 2^x \Rightarrow F_f \in \text{HE}$

$f \not\in \text{HI} \Rightarrow \forall x \ [ F_f(x) \downarrow \Rightarrow F_f(x) > 2^x ] \Rightarrow F_f \not\in \text{HE}$

Thus, $\text{HI} \leq_m \text{HE}$
HasExp(HE) = \{ f \mid \exists x \ f(x) \downarrow \ & \ f(x) = 2^x \}

Show that HE is many-one reducible to
HI = \{ f \mid \exists x \ f(x) \downarrow \ & \ f(x) = x \}

Let f be an arbitrary index

From f, define \( \forall x \ F_f(x) = | f(x) - 2^x | + x \)

\( f \in HE \Rightarrow \exists x \ F_f(x) = x \Rightarrow F_f \in HI \)

\( f \notin HE \Rightarrow \forall x [ F_f(x) \downarrow \Rightarrow F_f(x) > x ] \Rightarrow F_f \notin HE \)

Thus, HE \leq_m HI
Show a minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of \( \text{IE} \).

\[ \forall x \exists t [ \text{STP}(f,x,t) \& (\text{VALUE}(f,x,t) = 2^x) ] \]

\text{AD} \text{ looks to be up there with } \text{TOT}
IsExp(IE)=\{ f \mid \forall x \ f(x) \downarrow \ & \ f(x)=2^x \ \}\}

Use Rice’s Theorem to prove that IE is undecidable. Be Complete.
IE is non-trivial as \text{Power}(x) = 2^x \in IE \text{ and } \text{Co}(x) = 0 \notin IE
Let f,g be two arbitrary indices of procedures such that \forall x \ f(x) = g(x)

f \in IE \iff \forall x \ [ f(x) \downarrow \ & \ f(x) = 2^x ]
\iff \forall x \ [ g(x) \downarrow \ & \ g(x) = 2^x ]
as \forall x \ f(x) = g(x) \text{ and so } g \text{ has same I/O properties}
\iff g \in IE
IsExp(IE) = \{ f \mid \forall x \ f(x) \downarrow \ \& \ f(x) = 2^x \} \\

Show that \ TOT = \{ f \mid \forall x \ f(x) \downarrow \} \ is \ many-one reducible \ to \ IE. \\
Let \ f \ be \ an \ arbitrary \ index \\
From \ f, \ define \ \forall x \ F_f(x) = f(x) - f(x) + 2^x \\
f \in \ TOT \ \Rightarrow \ \forall x \ F_f(x) = 2^x \ \Rightarrow \ F_f \in \ IE \\
f \notin \ TOT \ \Rightarrow \ \exists x \ F_f(x) \uparrow \ \Rightarrow \ F_f \notin \ IE \\
Thus, \ TOT \leq_m IE
IsExp(IE) = \{ f \mid \forall x \ f(x) \downarrow \land f(x) = 2^x \}

Show that IE is many-one reducible to TOT = \{ f \mid \forall x \ f(x) \downarrow \}

Let f be an arbitrary index.
From f, define \forall x \ F_f(x) = \exists y \ [ f(x) = 2^x ]
If \ f \in IE \Rightarrow \forall x \ F_f(x) \downarrow \land F_f(x) = 1 \Rightarrow F_f \in TOT
If \ f \notin IE \Rightarrow \exists x \ F_f(x) \uparrow \Rightarrow F_f \notin TOT.
Thus, IE \leq_m TOT

That \ F_f(x) = 1 \ is not needed but gives more detail.
Note: It is rare to use \exists without using STP but I am fine with the search failing as a result of f diverging on some x or failing in a never-ending search.