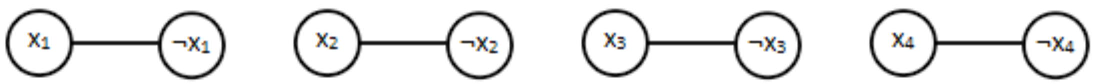
**COT 6410 Assignment 6 Sample Key**

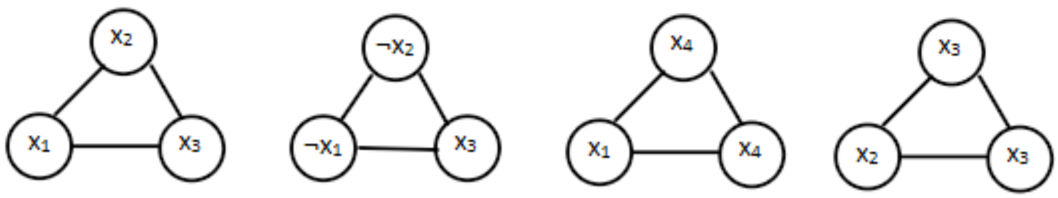
For the 3SAT instance: $(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor x_3) \land (x_1 \lor x_4 \lor x_4) \land (x_2 \lor x_3 \lor x_3)$:

**1.** The equivalent Vertex Cover instance:

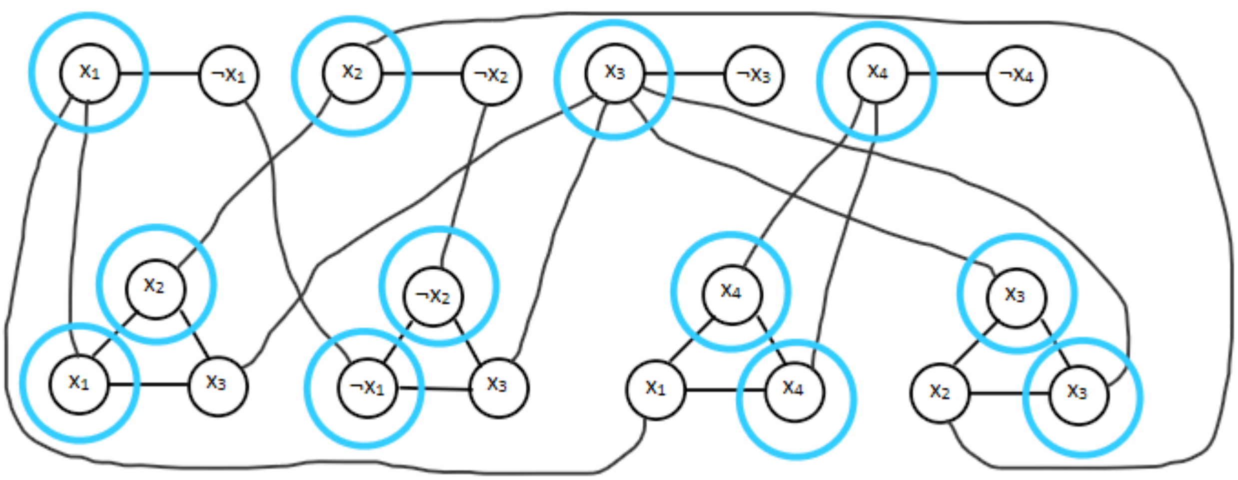
Variable gadgets:




Clause gadgets:




Combined gadgets:

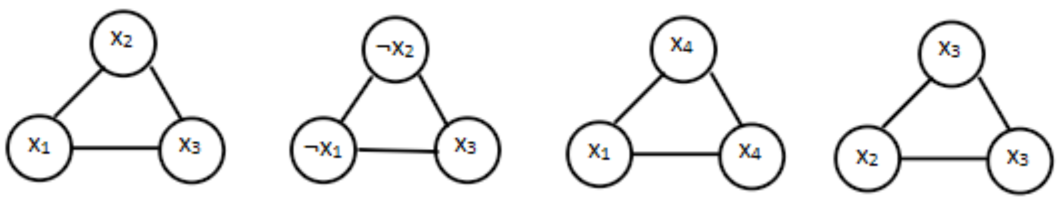



The number of vertices needed to be selected is $k = n + 2m = 4$ (the number of variables) $+ 2 \times 4$ (the number of clauses) $= 12$. Since the graph above has a vertex cover with exact 12 vertices (the circled ones), all clauses are satisfied.

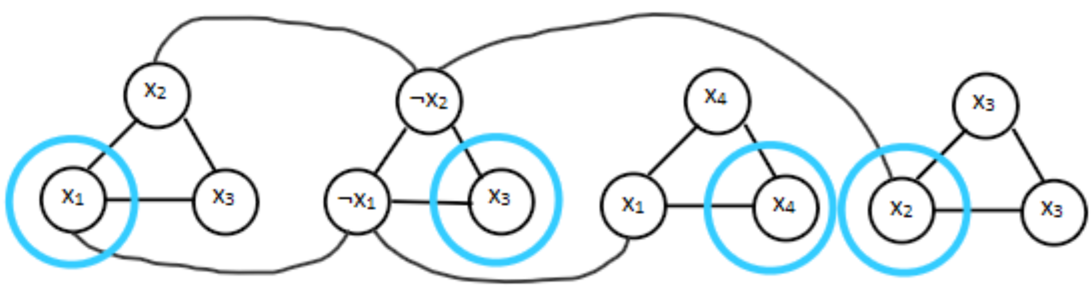
For the 3SAT instance: $(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_4 \vee x_4) \wedge (x_2 \vee x_3 \vee x_3)$:

**2.** The equivalent Independent Set instance:

Clause gadgets:




Combined gadgets:



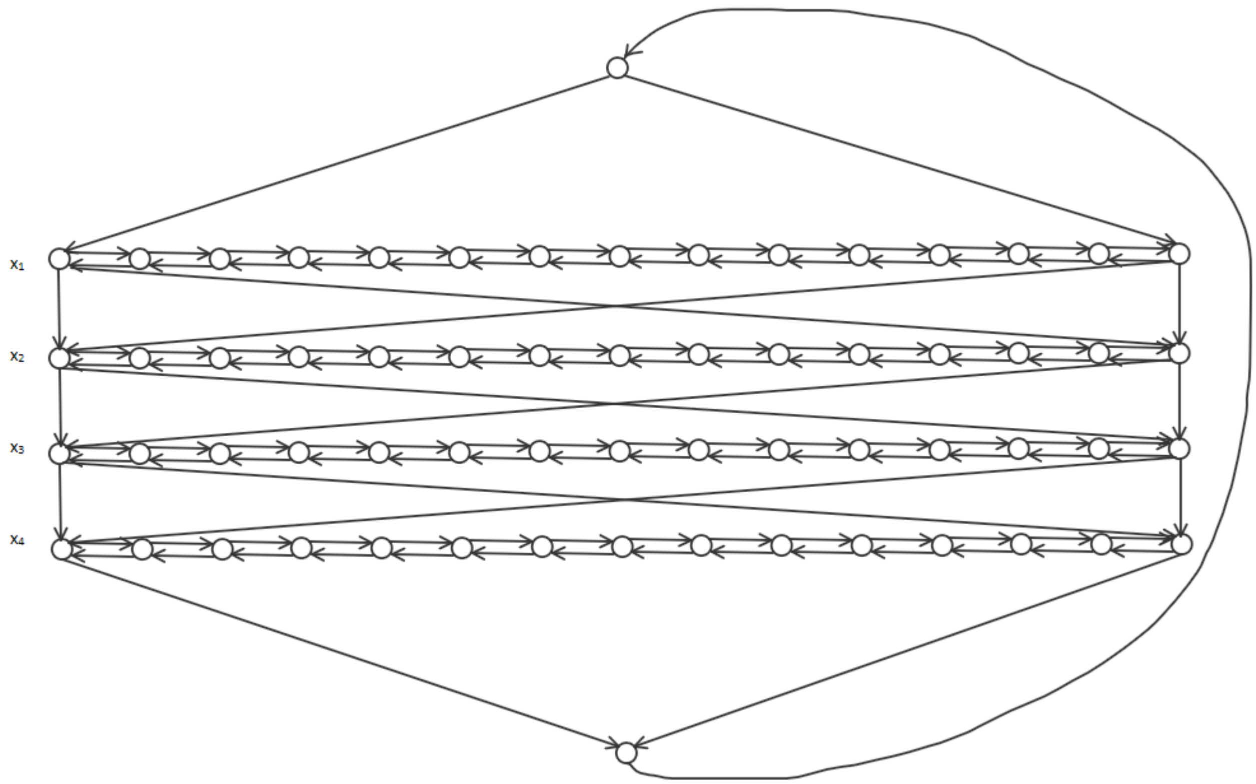

The number of vertices needed to be selected in the independent set is k = m = 4 (the number of clauses). Since the graph above has an independent set with exact 4 vertices (the circled ones), all clauses are satisfied.

For the 3SAT instance: $(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_4 \vee x_4) \wedge (x_2 \vee x_3 \vee x_3)$:

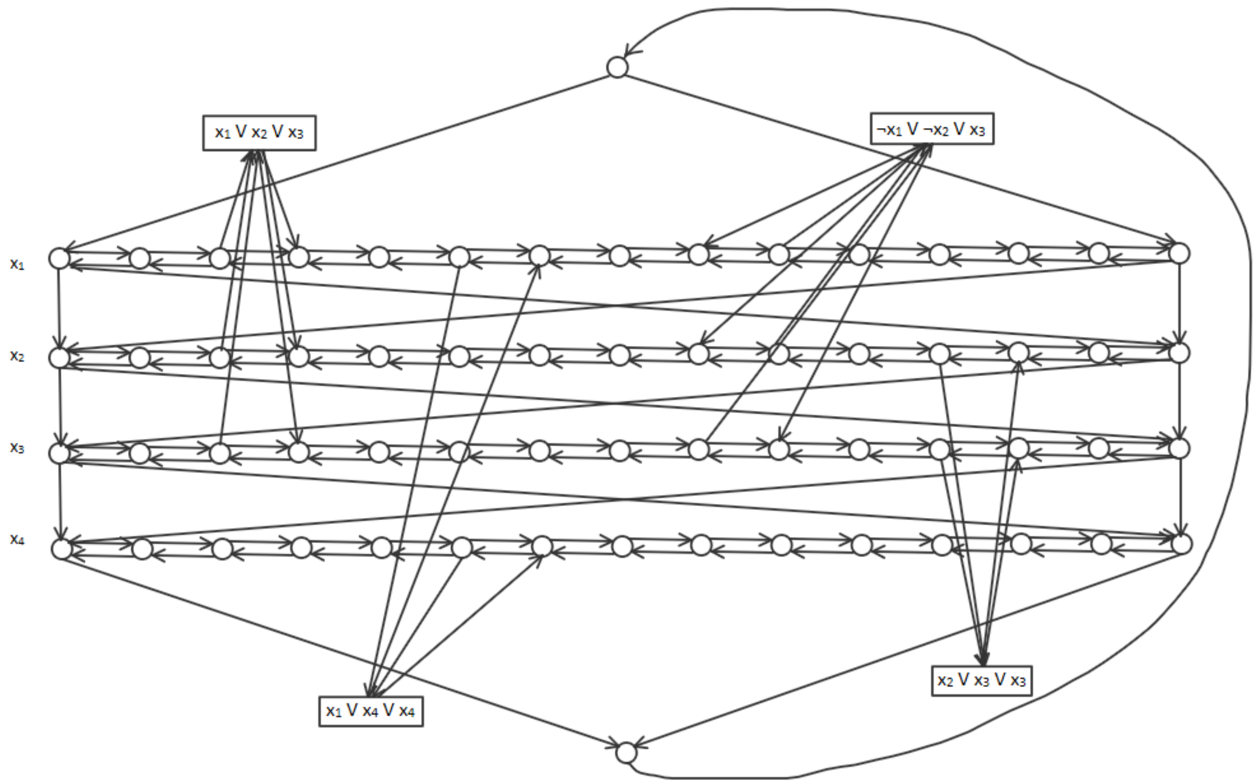
**3.** The equivalent Hamiltonian Circuit instance:

Assume for each path i has 3m + 3 vertices (i.e. vertex 1, vertex 2, …, vertex 3m + 3), where m is the number of clauses. If variable $x_i$ is True, the direction of passing the path i is left to right. If variable $\neg x_i$ is True, the direction of passing the path i is right to left. For clause $C_j$, if $x_i$ is in $C_j$, $C_j$, has edge from vertex 3j to vertex 3j + 1; if $\neg x_i$ is in $C_j$, $C_j$, has edge from vertex 3j + 1 to vertex 3j.

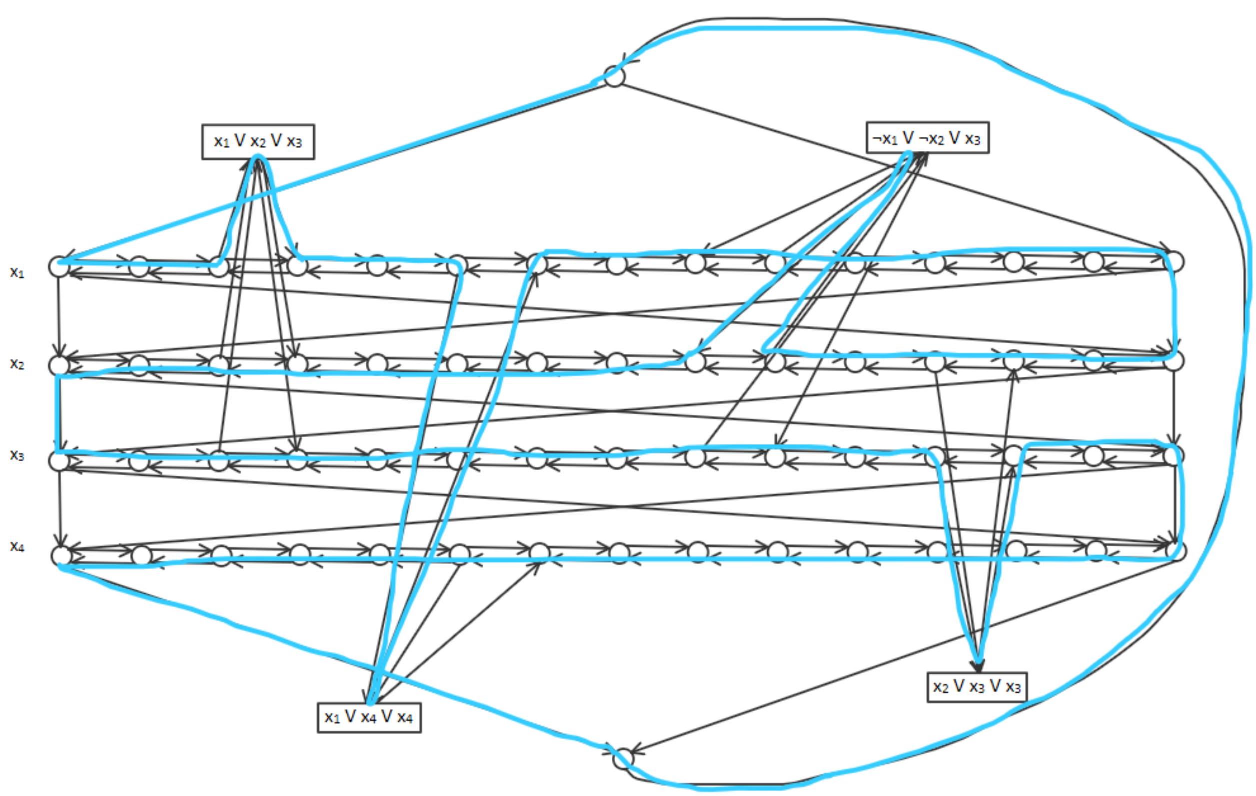
Variable gadgets:



Combined gadgets:




Below is the graph with a Hamiltonian Circuit highlighted, indicating all clauses are satisfied:



**4.** Consider the following set of independent tasks with associated task times:
**(T1,4), (T2,5), (T3,2), (T4,7), (T5,1), (T6,4), (T7,8)**
Fill in the schedules for these tasks under the associated strategies below.

Greedy using the list order above:

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *T1* | *T1* | *T1* | *T1* | *T3* | *T3* | *T5* | *T6* | *T6* | *T6* | *T6* | *T7* | *T7* | *T7* | *T7* | *T7* | *T7* | *T7* | *T7* |
| *T2* | *T2* | *T2* | *T2* | *T2* | *T4* | *T4* | *T4* | *T4* | *T4* | *T4* | *T4* | | | | | | | |

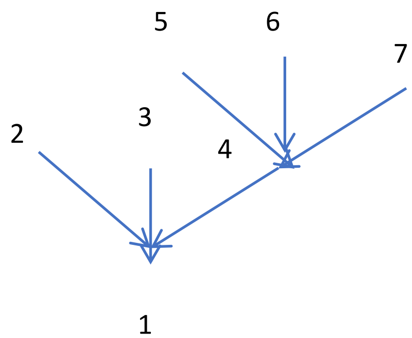
***19 units***

Greedy using a reordering of the list so that longest running tasks appear earliest in the list:

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *T7* | *T7* | *T7* | *T7* | *T7* | *T7* | *T7* | *T7* | *T1* | *T1* | *T1* | *T1* | *T6* | *T6* | *T6* | *T6* | | | |
| *T4* | *T4* | *T4* | *T4* | *T4* | *T4* | *T4* | *T2* | *T2* | *T2* | *T2* | *T2* | *T3* | *T3* | *T5* | | | | |

***16 units (optimal)***

**5.** Consider a very simple unit execution time tree with just 7 tasks that we wish to schedule on 2 processors. The tree is below.




**a.)** Show the Gantt chart associated with the optimal schedule based on the assigned priorities.

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***T7*** | ***T5*** | ***T4*** | ***T1*** | | | | | | | | | | | | | | | |
| ***T6*** | ***T3*** | ***T2*** | | | | | | | | | | | | | | | | |

**b.)** Show the Gantt chart associated with some optimal schedule when this is treated as an anti-tree (dependency arrows reversed).

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***T1*** | ***T4*** | ***T5*** | ***T7*** | | | | | | | | | | | | | | | |
| | ***T2*** | ***T3*** | ***T6*** | | | | | | | | | | | | | | | |

**c.)** Show the Gantt chart associated with the schedule of this anti-tree when inverted priorities are used (1 is highest, 2 is second highest, etc.). Comment on any observation you might have of this versus the schedule in **(b)**.

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***T1*** | ***T2*** | ***T4*** | ***T5*** | ***T7*** | | | | | | | | | | | | | | |
| | ***T3*** | | ***T6*** | | | | | | | | | | | | | | | |

***This schedule ignores the importance of completing T4 to open up T5, T6 and T7. In other words, it is not cognizant of the importance of critical paths.***