Assignment#4 Sample Key

1. NotDominating(ND) = { f | for some x, f(x) < x }.

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **ND**.

∃<x,t> [STP(f,x,t) & (VALUE(f,x,t) < x)]}

b.) Use Rice's Theorem to prove that **ND** is undecidable. Be Complete.

ND is non-trivial as **C0(x) = 0** ∈ **ND** and **S(x) = x+1** ∉ **ND**

Let f,g be two arbitrary indices of procedures such that ∀**x f(x) = g(x)**

f ∈ **ND** iff **∃x f(x) < x iff $f(x_0) < x_0$ for some x_0 iff $g(x_0) < x_0$ as ∀x f(x) = g(x) implies ∃x g(x) < x iff g** ∈ **ND**

f ∉ **ND** iff ∀**x [f(x)↓ implies f(x)>x] iff** ∀**x [g(x)↓ implies g(x)>x] as** ∀**x f(x) = g(x) iff g** ∉ **ND**

c.) Show that **K = { f | f(f)** converges **}** is many-one reducible to **ND**.

Let **f** be an arbitrary index. From **f**, define ∀**x F_f(x) = f(f)-f(f)**.
f ∈ **K** implies ∀**x F_f(x) = 0** implies **F_f** ∈ **ND**.
f ∉ **K** implies ∀**x F_f(x)** diverges implies **F_f** ∉ **ND**.

Thus, **K $\leq_m$ ND**

d.) Show that **ND** is many-one reducible to **K = { f | f(f)** converges **}**

Let **f** be an arbitrary index. From **f**, define ∀**y F_f(y) = ∃<x,t> STP(f,x,t) & (VALUE(f,x,t) < x)**
f ∈ **ND** implies ∀**y F_f(y)** converges implies **F_f(F_f)** converges implies **F_f** ∈ **K**
f ∉ **ND** implies ∀**y F_f(y)** diverges implies **F_f** ∉ **K**.

Thus, **ND $\leq_m$ K**

2. AlwaysDominates(AD) = { f | for all x, f(x) > x }

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **AD**.

$\forall$x $\exists$t [STP(f,x,t) & (VALUE(f,x,t) > x)]}

b.) Use Rice's Theorem to prove that **AD** is undecidable. Be Complete.

AD is non-trivial as **S(x) = x+1** $\in$ **AD** and **C0(x) = 0** $\notin$ **AD**

Let f,g be two arbitrary indices of procedures such that $\forall$x f(x) = g(x)

f $\in$ **AD** iff **$\forall$x f(x) < x iff $\forall$x g(x) < x iff g** $\in$ **ND**

c.) Show that **TOT = { f |** for all **x**, f**(x)** converges **}** is many-one reducible to **AD**.

Let **f** be an arbitrary index. From **f**, define **$\forall$x F_f(x) = f(x)-f(x) + x + 1**.
f $\in$ **TOT** implies **$\forall$x F_f(x) = x+1** implies **F_f** $\in$ **AD.**
f $\notin$ **TOT** implies **$\exists$x F_f(x)** diverges implies **F_f** $\notin$ **AD**.

Thus, **TOT $\leq_m$ AD**

d.) Show that **AD** is many-one reducible to **TOT = { f |** for all **x**, f**(x)** converges **}**

Let **f** be an arbitrary index. From **f**, define **$\forall$x F_f(x) = μy [f(x) > x]**
f $\in$ **AD** implies **$\forall$x F_f(x)** converges implies **F_f** $\in$ **TOT**
f $\notin$ **AD** implies **$\exists$x F_f(x)** diverges implies **F_f** $\notin$ **TOT**.

Thus, **AD $\leq_m$ TOT**