

Consider the 3SAT instance:

$(x_1 \vee \neg x_2 \vee \neg x_3 \vee \neg x_4 \vee x_5) \wedge (\neg x_1 \vee x_2 \vee x_3 \vee x_4) \wedge (x_1 \vee \neg x_4) \wedge (x_3)$

1. Recast this as an instance of 3SAT.

ANS:  $(x_1 \vee \neg x_2 \vee x_6) \wedge (\neg x_3 \vee \neg x_4 \vee x_7) \wedge (x_5 \vee \neg x_6 \vee \neg x_7) \wedge (\neg x_1 \vee x_2 \vee x_8) \wedge (x_3 \vee x_4 \vee \neg x_8) \wedge (x_1 \vee \neg x_4 \vee x_1) \wedge (x_3 \vee x_3 \vee x_3)$

2. Construct the SubsetSum instance equivalent to this and state what rows must be chosen to achieve the desired sum.

ANS:

$c_1 = (x_1 \vee \neg x_2 \vee x_6)$

$c_2 = (\neg x_3 \vee \neg x_4 \vee x_7)$

$c_3 = (x_5 \vee \neg x_6 \vee \neg x_7)$

$c_4 = (\neg x_1 \vee x_2 \vee x_8)$

$c_5 = (x_3 \vee x_4 \vee \neg x_8)$

$c_6 = (x_1 \vee \neg x_4 \vee x_1)$

$c_7 = (x_3 \vee x_3 \vee x_3)$

	x1	x2	x3	x4	x5	x6	x7	x8	C1	C2	C3	C4	C5	C6	C7
x1	1	0	0	0	0	0	0	0	1	0	0	0	0	1	0
~x1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
x2	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0
~x2	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
x3	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1
~x3	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
x4	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0
~x4	0	0	0	1	0	0	0	0	0	1	0	0	0	1	0
x5	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0
~x5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
x6	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
~x6	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
x7	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
~x7	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0
x8	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0
~x8	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0
C1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
C1'	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
C2	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
C2'	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
C3	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
C3'	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
C4	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
C4'	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
C5	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C5'	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
C6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
C6'	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
C7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
C7'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
	1	1	1	1	1	1	1	1	3	3	3	3	3	3	3

3. Recast the SubsetSum instance in Part 2 as a Partition instance (really easy). Show the Partitioning into equal subsets.

Ans:

G = 111111113333333

sum= 2222222555555

2 \* sum - G = 3333333777777

sum + G = 3333333888888

sum is the sum of all rows.

The partitions are as follows:

Partition 1:

33333333	77777777	$2 * \text{sum} - G$
10000000	1000010	x1
01000000	0001000	x2
00100000	0000101	x3
00010000	0000100	x4
00001000	0010000	x5
00000100	0010000	$\sim x6$
00000010	0100000	x7
00000001	0001000	x8
00000000	1000000	C1
00000000	1000000	C1'
00000000	0100000	C2
00000000	0100000	C2'
00000000	0010000	C3
00000000	0001000	C4
00000000	0000100	C5
00000000	0000010	C6
00000000	0000010	C6'
00000000	0000001	C7
00000000	0000001	C7'

Partition 2:

33333333	88888888	sum+G
10000000	0001000	~x1
01000000	1000000	~x2
00100000	0100000	~x3
00010000	0100010	~x4
00001000	0000000	~x5
00000100	1000000	x6
00000010	0010000	~x7
00000001	0000100	~x8
00000000	0010000	C3'
00000000	0001000	C4'
00000000	0000100	C5'

4. For the 0-1 Integer Linear Programming instance:

ANS:

Assume  $0 \leq x_1, x_2, x_3, x_4, x_5 \leq 1$

$$x_1 + (1-x_2) + (1-x_3) + (1-x_4) + x_5 \geq 1$$

$$(1-x_1) + x_2 + x_3 + x_4 \geq 1$$

$$x_1 + (1-x_4) \geq 1$$

$$x_3 = 1$$

We choose:  $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 0, x_5 = 1$