Assignment#4 Key

1. HasDup(ND) = { f | for some x,y, x≠y, f(x) = f(y) }

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **ND**.

$\exists\langle x,y,t\rangle$ [STP(f,x,t) & STP(f,y,t) & x≠y & (VALUE(f,x,t) = VALUE(f,xyt))]

b.) Use Rice's Theorem to prove that **ND** is undecidable. Be Complete.

ND is non-trivial as **C0(x) = 0** $\in$ **ND** and **S(x) = x+1** $\notin$ **ND**

Let f,g be two arbitrary indices of procedures such that $\forall x$ f(x) = g(x)

f $\in$ ND iff **$\exists x,y$ [(x≠y) & f(x) = f(y)) iff $f(x_0) = f(y_0)$ for some x_0, y_0 iff $g(x_0) = g(y_0)$ as $\forall x$ f(x) = g(x) implies $\exists x,y$ [(x≠y) & g(x) = g(y)) iff g $\in$ ND**

f $\notin$ ND iff **$\forall x,y$ [(x≠y) implies f(x) ≠ f(y)) iff $\forall x,y$ [(x≠y) implies g(x) ≠ g(y)) as $\forall x$ f(x) = g(x) iff g $\notin$ ND**

c.) Show that **K = { f | f(f)** converges **}** is many-one reducible to **ND**.

Let **f** be an arbitrary index. From **f**, define $\forall$**x $F_f(x)$ = f(f)-f(f)**.
f $\in$ **K** implies $\forall$**x $F_f(x)$ = 0** implies **F_f** $\in$ **ND**.
f $\notin$ **K** implies $\forall$**x $F_f(x)\uparrow$** implies **F_f** $\notin$ **ND**.

Thus, **K $\leq_m$ ND**

d.) Show that **ND** is many-one reducible to **K = { f | f(f)** converges **}**

Let **f** be an arbitrary index. From **f**, define $\forall$**y $F_f(y)$ = $\exists\langle x,y,t\rangle$ [STP(f,x,t) & STP(f,y,t) & x≠y & (VALUE(f,x,t) = VALUE(f,xyt))]**
f $\in$ ND implies $\forall$**y $F_f(y)\downarrow$** implies **$F_f(F_f)$** converges implies **F_f** $\in$ **K**
f $\notin$ ND implies $\forall$**y $F_f(y)\uparrow$** implies **F_f** $\notin$ **K**.

Thus, **ND $\leq_m$ K**

2. AlwaysDominates(AD) = { f | for all x, f(x) > x }

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **AD**.

$\forall x\, \exists t\, [STP(f,x,t)\ \&\ (VALUE(f,x,t) > x)]\}$

b.) Use Rice's Theorem to prove that **AD** is undecidable. Be Complete.

AD is non-trivial as $S(x) = x+1 \in AD$ and $C0(x) = 0 \notin AD$

Let f,g be two arbitrary indices of procedures such that $\forall x\ f(x) = g(x)$

$f \in AD$ iff $\forall x\ f(x) > x$ iff $\forall x\ g(x) > x$ iff $g \in ND$

c.) Show that **TOT = { f |** for all **x**, f**(x)** converges **}** is many-one reducible to **AD**.

Let **f** be an arbitrary index. From **f**, define $\forall x\ F_f(x) = f(x)-f(x) + x + 1$.
$f \in TOT$ implies $\forall x\ F_f(x) = x+1$ implies $F_f \in AD$.
$f \notin TOT$ implies $\exists x\ F_f(x)\downarrow$ implies $F_f \notin AD$.

Thus, $TOT \leq_m AD$

d.) Show that **AD** is many-one reducible to **TOT = { f |** for all **x**, f**(x)** converges **}**

Let **f** be an arbitrary index. From **f**, define $\forall x\ F_f(x) = \exists y\ [\ f(x) > x\ ]$
$f \in AD$ implies $\forall x\ F_f(x)\downarrow$ implies $F_f \in TOT$
$f \notin AD$ implies $\exists x\ F_f(x)\uparrow$ implies $F_f \notin TOT$.

Thus, $AD \leq_m TOT$