Nondeterminism

An NFA looks like a DFA, *except* an NFA:
- Can have more than one possible transition per state per input symbol
- Doesn't have to have a transition for every state for every input symbol
- Can transition on the empty string

It also *works* like a DFA, except acceptance is nondeterministic
- A DFA accepts if *the* path for the input string ends on an accept symbol
- An NFA accepts if *any* path for the input string ends on an accept symbol
The easiest way to think of how an NFA works is to think of a *threaded DFA*:

- Every time there is a choice of more than one path, the NFA splits off a copy of itself to follow each path.
- The copies conceptually run in parallel.
- A copy that reaches the end of input either accepts or rejects normally.
- A copy that reaches a symbol it cannot transition on stops and rejects.
- The NFA itself accepts if *any* copy accepts.
Input 010110 on our NFA
NFA Example

What does this machine do?
What does this machine do?

This machine accepts all strings consisting of a number of zeroes that's a multiple of either 2 or 3

- It's like our modulus machine from last lecture, except it accepts $x \mod 2$ and $x \mod 3$ at the same time
An NFA and its DFA
(Yes, this is, in fact, the best we can do.)
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No, you’re not missing some kind of subtle elegance.
This DFA is a hideous monster.
That’s why we have NFAs.
Definition: Nondeterministic Finite Automaton

A **nondeterministic finite automaton** is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\) that consists of:

- \(Q\) - A finite set of states
- \(\Sigma\) - An alphabet
- \(\delta: Q \times \Sigma \rightarrow P(Q)\) - A transition function
- \(q_0 \in Q\) - A start state
- \(F \subseteq Q\) - A set of accept (or final) states
Equivalence

The capabilities of NFAs are a strict superset of the capabilities of DFAs, so every DFA can obviously be made into an NFA.

- The reverse is less obvious – but is nonetheless true
- ...and immensely important, as we'll get to soon enough
Proof: NFA/DFA Equivalence

Prove that every NFA has an equivalent DFA.

(Board work: Theorem 1.39)
Example: NFA/DFA Equivalence

(Board work: Example 1.41)
Consequences 1

If every NFA has a DFA, then every language that can be recognized with an NFA can be recognized with a DFA.

But that means...
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But that means...

Corollary to NFA/DFA Equivalence: A language is regular if and only if it can be recognized by an NFA.
Consequences 2

We ended last session by taking about 20 minutes to prove that the class of regular languages was closed under union

- Let's do that again, a lot faster
- After that, we'll prove that it's closed under concatenation and star closure

(Board work: Theorems 1.45, 1.47, 1.49)
Next Time:
Regular Expressions