Generalized Minimum Clique Problem (GMCP)

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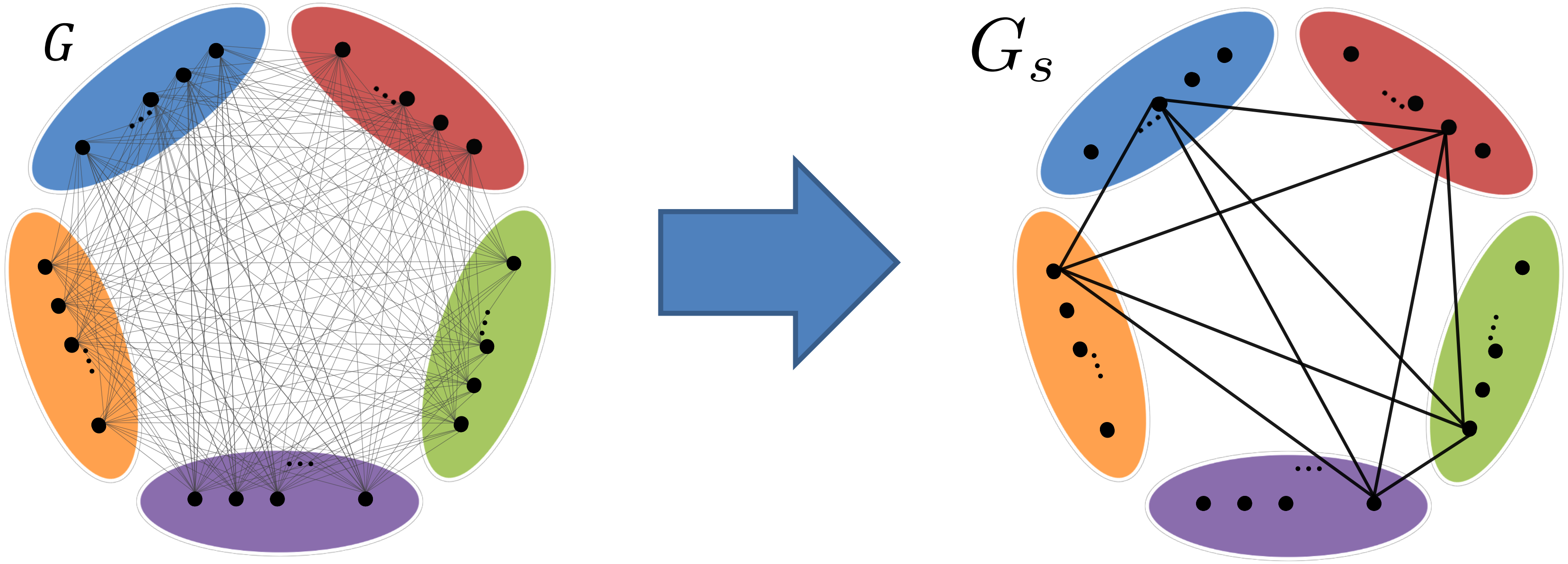
- **Traveling Salesman** : find the minimal cycle which visits all the nodes exactly once
- **Generalized Traveling Salesman** : Find the minimal cycle which connects all the clusters while exactly one node from each is visited
- **GMCP**: Find a subset of the nodes that includes exactly one node from each cluster while the cost of the complete graph that the subset forms is minimized (Can be reduced to Traveling Salesman, therefore its NP-Hard)



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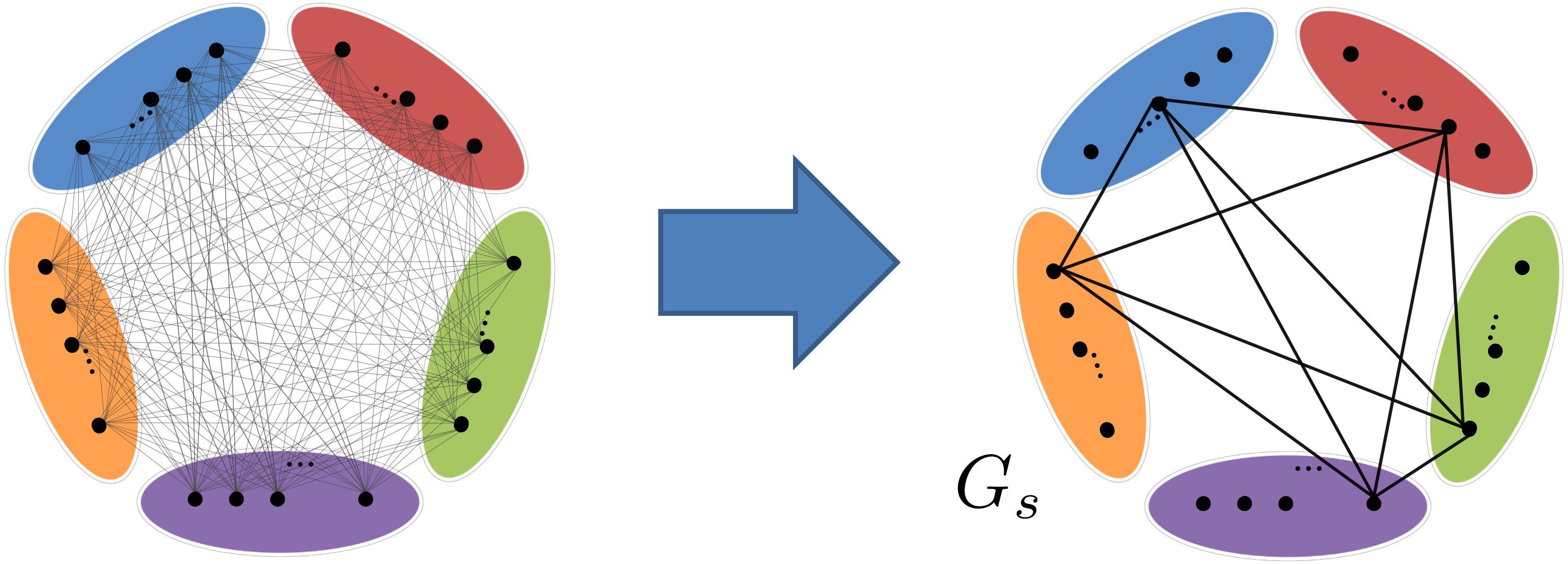


$$G = (V, E, w)$$

V Nodes E Edges w Edge Weights

$$G_s = (V_s, E_s, w_s)$$

$$\hat{\mathbf{V}}_\mathbf{s} = argmin_{\mathbf{V_s}} C(\mathbf{V_s})$$

Approximate Solution
(Heuristic)

An Approximate Solution for GMCP

- Best solution is initialized by picking one random node from each cluster
  - Complexity O(L) for verification where L : number of clusters
- Fix neighborhood size to 1 and identify the δ best solutions
  - Complexity O((K-1)L), where K : number of nodes
- At each iteration neighbor solutions of the size |V|-δ to |V|-1 are evaluated and compared with the Best solution
  - Complexity $O(K^{\delta}L)$
- Iterations continue until either convergence criteria(Minimum found) or termination criteria(Maximum time/Maximum iterations) are met
- Complexity of the whole process :
  - $O(L) \times O((K-1)L) + O(K^{\delta}L) = O((K-1)L^2) + O(K^{\delta}L) = O(KL^2 + K^{\delta}L)$ ➔ Polynomial Time

Exact Solution
(BIP)

Solving GMCP using Binary Integer Programing

- Finds the optimal solution

$$\begin{cases} W^T X \\ AX = B \\ MX \le N \end{cases}$$

- W Is the object weights
- X Binary variable for the nodes and edges

- Constraints ensure the solution is a valid GMCP instance

$$AX = B$$

$$MX \le N$$

Constraints

Ensures that one and only one node is selected from each cluster

$$\{\forall j | 1 \leq j \leq h\} : \sum_{i=1}^{k} \nu_i^j = 1.$$

states if one node is selected to be in Gs, then exactly (h − 1) of its edges should be included in Gs

$$\{\forall m, n | 1 \leq m \leq h, 1 \leq n \leq k\} : \sum_{i=1}^{h} \sum_{j=1}^{k} \varepsilon_{mn}^{ij} = \nu_n^m.(h-1).$$

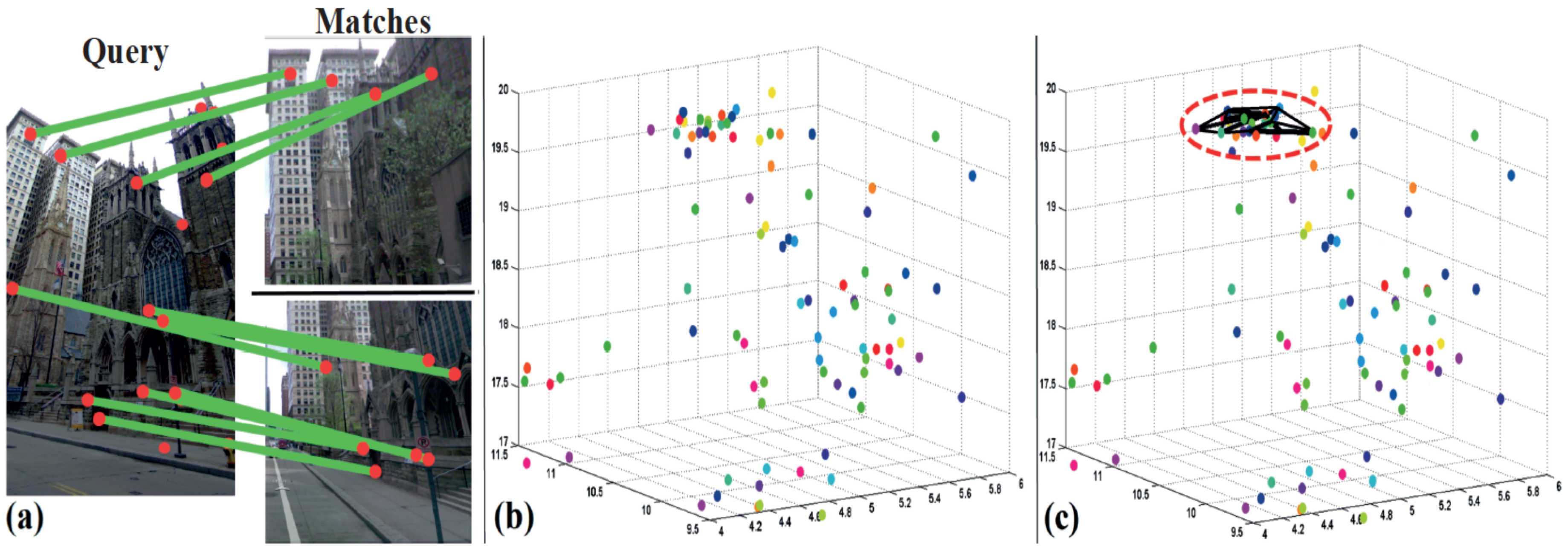
ensures if one edge is included in Gs, then the variables of its parent nodes have to be 1 and vice versa

$$\{\forall m, n, i, j | 1 \leq m, j \leq h, 1 \leq n, i \leq k\} : \nu_n^m \wedge \nu_i^j = \varepsilon_{mn}^{ji}.$$

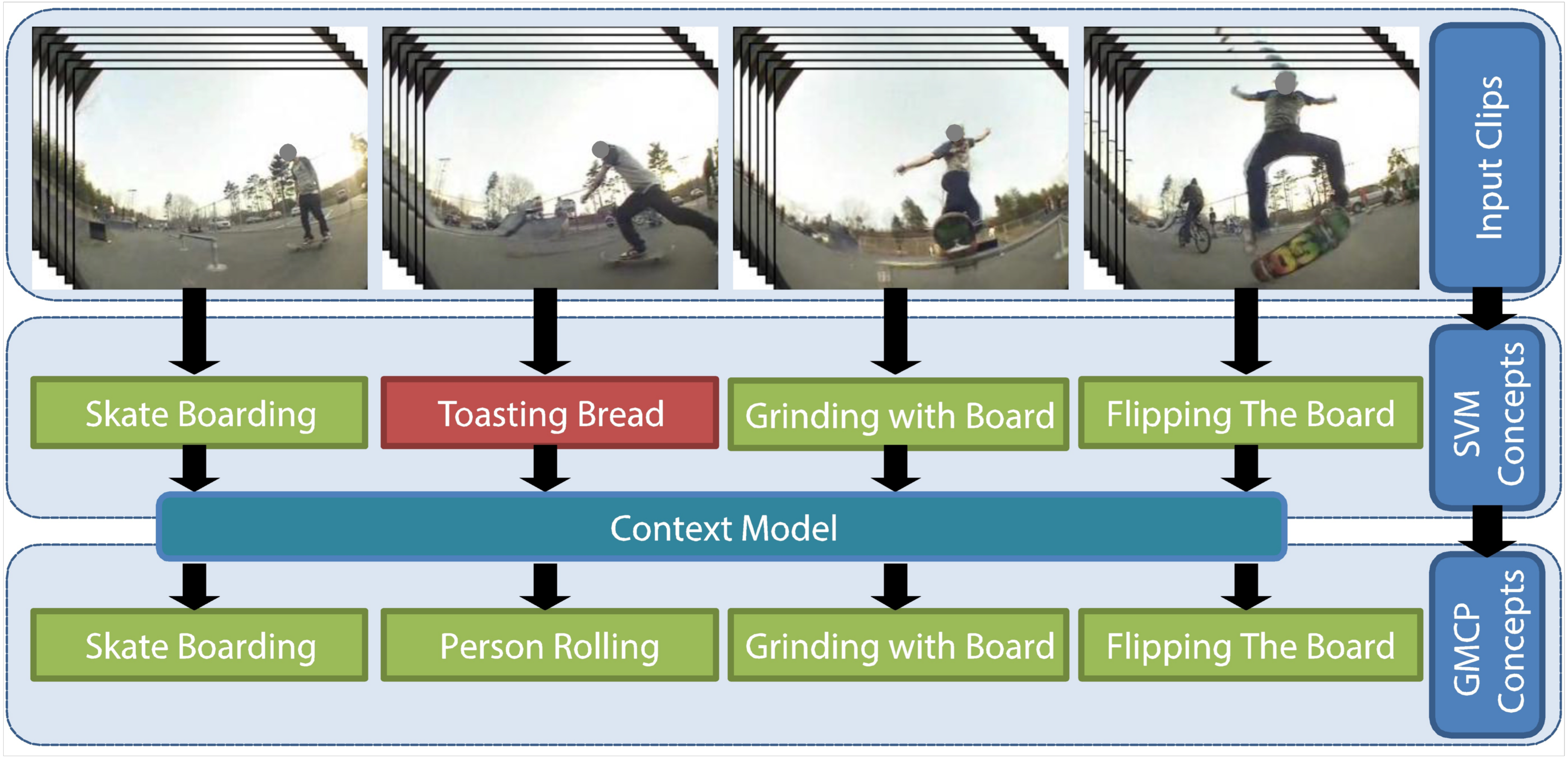
Applications

Problem : Image Geo-Localization

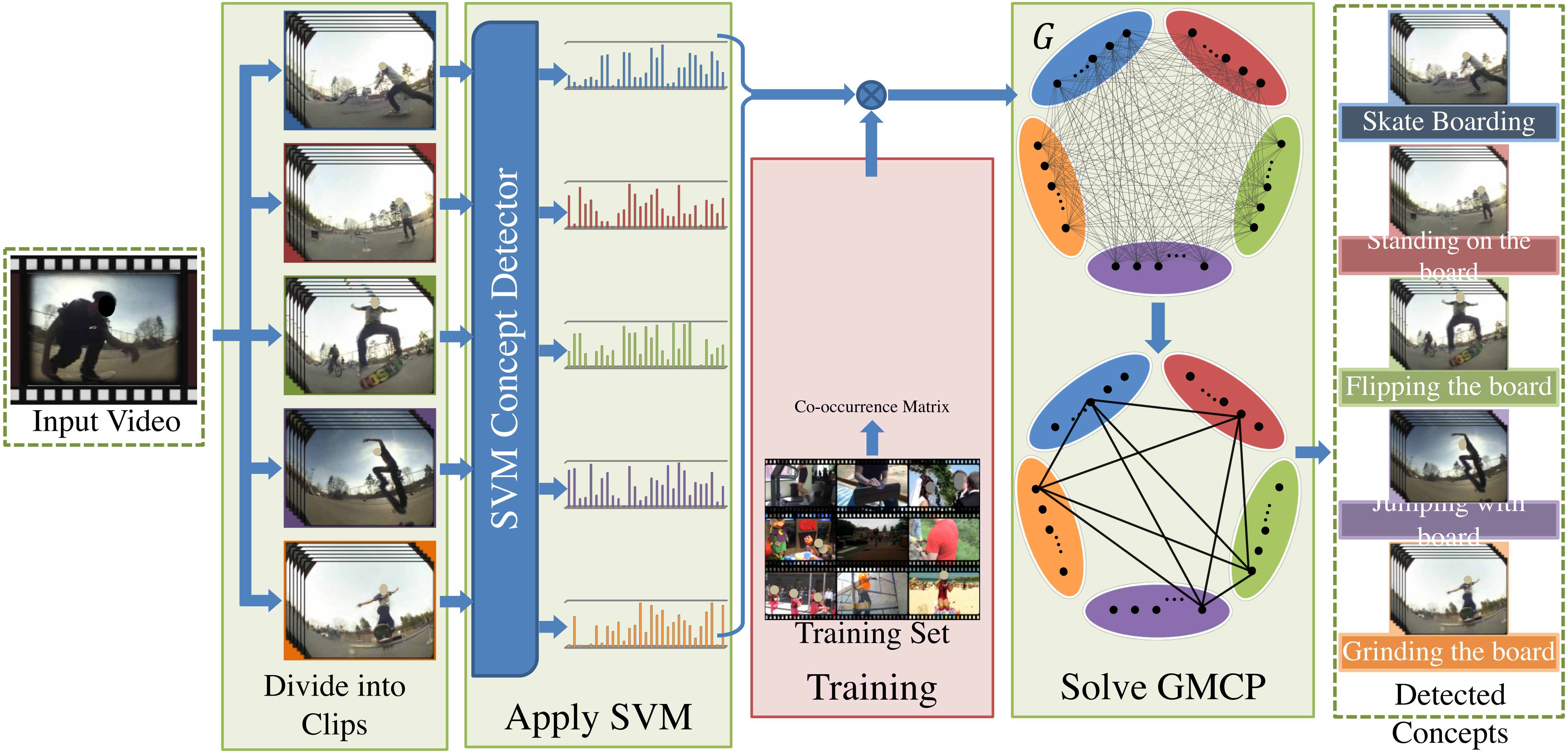
Query Image → Local Feature Extraction → Retrieving kNNs for each query feature → Multiple NN pruning → GMCP-based feature matching → Geo-localization using voting scheme



Problem : Concept Detection in Complex Videos



Problem : Concept Detection in Complex Videos



Thanks

Questions??

- In BIP solution for GMCP there is a Boolean variable associated to each node and each edges. What happens if we allow the solver to pick any number between [0 1] instead of a binary variable 0 and 1?
- Show that GMCP is NP-HARD (Show how does it reduce to TSP)?