

1. Let set **A** be **infinite recursive**, **B** be **re non-recursive** and **C** be **non-re**. Using the terminology **(REC) recursive**, **(RE) re, non-recursive**, **(NR) non-re (possibly co-re)**, categorize each set by dealing with the cases I present, saying whether or not the set can be of the given category and briefly, but convincingly, justifying each answer (BE COMPLETE). You may assume sets like **N** are infinite REC; **K** and **K₀** are RE; and **TOTAL** is non-re. You may also assume, for any set **S**, the existence of comparably hard sets

$$S_E = \{2x | x \in S\} \text{ and } S_D = \{2x+1 | x \in S\}.$$

a.) $A + B = \{ x | x = y + z, \text{ for some } y \in A \text{ and some } z \in B \}$

REC:

b.) $A \cap C = \{ x | x \in A \text{ and } x \in C \text{ and } x \notin A \}$

RE:

2. Choosing from among **(REC) recursive**, **(RE) re non-recursive**, **(coRE) co-re non-recursive**, **(NRNC) non-re/non-co-re**, categorize each of the sets in a) through d). Justify your answer by showing some minimal quantification of some known recursive predicate.

a) $A = \{ \langle f, g \rangle | \exists x \varphi_f(x) \downarrow \text{ and } \varphi_g(x) = \varphi_f(x) \}$.

b.) $B = \{ f | \text{range}(\varphi_f) \text{ is empty} \}$

c.) $C = \{ \langle f, x \rangle | \varphi_f(x) \downarrow \text{ but takes at least 10 steps to do so} \}$

d.) $D = \{ f | \varphi_f \text{ diverges for some value of } x \}$

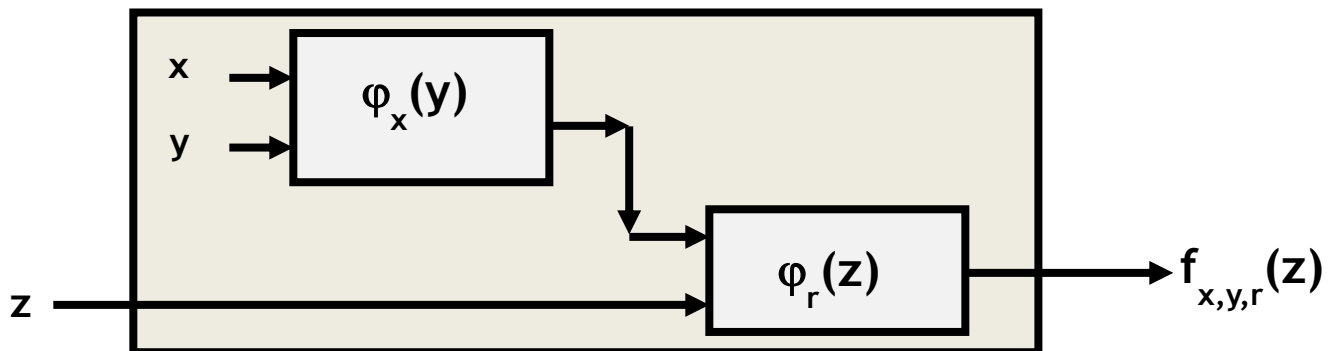
3. Looking back at Question 1, which of these are candidates for using Rice's Theorem to show their unsolvability? Check all for which Rice Theorem might apply.

a) _____ b) _____ c) _____ d) _____

4. Let **S** be an arbitrary semi-decidable set. By definition, **S** is the domain of some partial recursive function **g_S**. Using **g_S**, constructively show that **S** is the range of some partial recursive function, **f_S**. No proof is required; just the construction is needed here.

5. Using the definition that S is recursively enumerable iff S is the range of some effective procedure f_S (partial recursive function), prove that if both S and its complement $\sim S$ are recursively enumerable (using semi-decision effective procedures f_S and $f_{\sim S}$) then S is decidable. To get full credit, you must show the characteristic function for S , χ_S , in all cases. Also, be sure to discuss why your χ_S works.

6. Rice's Theorem deals with attributes of certain types of problems P about partial recursive functions and their corresponding sets of indices S_P . The following image describing a function $f_{x,y,r}$ is central to understanding Rice's Theorem.



Explain the meaning of this by indicating:

- What assumption do we make about what kind of functions are not in P ?
- What is r , how is it chosen and how can we guarantee its existence?
- Using recursive function notations, write down precisely what $f_{x,y,r}$ computes for the Strong Form of Rice's Theorem.

How does this function $f_{x,y,r}$ behave with respect to x,y and r , and how does that relate to the original problem, P , and set, S_P ?

10. We described the proof that 3SAT is polynomial reducible to Subset-Sum. You must repeat that.

a.) Assuming a 3SAT expression $(a + a + \sim b) (\sim a + b + c)$, fill in all omitted values (zeroes elements can be left as omitted) of the reduction from 3SAT to Subset-Sum.

	a	b	c	$a + a + \sim b$	$\sim a + b + c$
a					
$\sim a$					
b					
$\sim b$					
c					
$\sim c$					
C1					
C1'					
C2					
C2'					
	1	1	1	3	3

b.) List some subset of the numbers above (each associated with a row) that sums to 1 1 1 3 3. Indicate what the related truth values are for a, b and c.

11. Present a gadget used in the reduction of 3-SAT to some graph theoretic problem where the gadget guarantees that each variable is assigned either True or False, but not both. Of course, you must tell me what graph theoretic problem is being shown NP-Complete and you must explain why the gadget works.

12. Let Q be some problem (an optimization or decision problem). Assuming $\leq p$ means many-one reducible in polynomial time and $\leq tp$ means Turing-reducible in polynomial time, categorize Q as being in one of P , NP , $co-NP$, NP -Complete, NP -Easy, NP -Hard, or NP -Equivalent (see first two pages for definitions of each of these concepts). For each case, choose the most precise category. I filled in one answer already.

Description of Q	Category
Q is decidable in deterministic polynomial time	
For some R in NP , $Q \leq tp R$	
Q is both NP -Easy and NP -Hard	
Q is in NP and if R is in NP then $R \leq p Q$	
A solution to Q is verifiable in deterministic polynomial time	
Q 's complement is in NP	

13. A graph G is k -Colorable if its vertices can be colored using just k (or fewer colors) such that adjacent vertices have different colors. The **Chromatic Number** of a graph G is the smallest number k for which G is k -Colorable. k -Colorable is a decision problem that has parameters (G, k) , whereas the **Chromatic Number** problem is a function with a single parameter G . In all cases, assume G has n vertices.

a.) Show that k -Colorable $\leq tp$ Chromatic Number ($\leq tp$ means Turing reducible in polynomial time).

b.) Show that Chromatic Number $\leq tp$ k -Colorable ($\leq tp$ means Turing reducible in polynomial time).

14. **Partition** refers to the decision problem as to whether some set of positive integers S can be partitioned into two disjoint subsets whose elements have equal sums. **Subset-Sum** refers to the decision problem as to whether there is a subset of some set of positive integers S that precisely sums to some goal number G .

a.) Show that **Partition** $\leq p$ **Subset-Sum**.

b.) Show that **Subset-Sum** $\leq p$ **Partition**.

15. **QSAT** is the decision problem to determine if an arbitrary fully quantified Boolean expression is true. Note: **SAT** only uses existential, whereas **QSAT** can have universal qualifiers as well so it includes checking for Tautologies as well as testing Satisfiability. What can you say about the complexity of **QSAT** (is it in **P**, **NP**, **NP-Complete**, **NP-Hard**)? Justify your conclusion.

16. Specify True (T) or False (F) for each statement.

Statement	T or F
Every Regular Language is also a Context Free Language	
Phrase Structured Languages are the same as RE Languages	
The Context Free Languages are closed under Complement	
A language is recursive iff it and its complement are re	
PCP is undecidable even for one letter systems	
Membership in Context Sensitive Languages is undecidable	
Every RE language is Turing reducible to its complement	
Emptiness is undecidable for Context Sensitive Languages	
The complement of a trace language is Context Free	
The word problem for two-letter Semi-Thue Systems is decidable	