COT 6410 Assignment 6

Solution

For the 3SAT instance: $(x_1 \vee x_2 \vee x_3) \wedge (\neg x_1 \vee \neg x_2 \vee x_3) \wedge (x_1 \vee x_4 \vee x_4) \wedge (x_2 \vee x_3 \vee x_3)$:

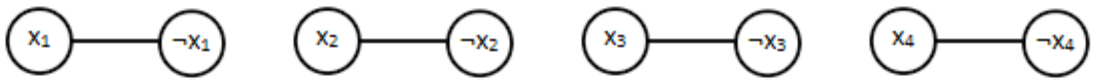
(1) The equivalent SubsetSum instance:

		x_1	x_2	x_3	x_4	$x_1 \vee x_2 \vee x_3$	$\neg x_1 \vee \neg x_2 \vee x_3$	$x_1 \vee x_4 \vee x_4$	$x_2 \vee x_3 \vee x_3$
1	x_1	1	0	0	0	1	0	1	0
2	$\neg x_1$	1	0	0	0	0	1	0	0
3	x_2	0	1	0	0	1	0	0	1
4	$\neg x_2$	0	1	0	0	0	1	0	0
5	x_3	0	0	1	0	1	1	0	2
6	$\neg x_3$	0	0	1	0	0	0	0	0
7	x_4	0	0	0	1	0	0	2	0
8	$\neg x_4$	0	0	0	1	0	0	0	0
9	C_1	0	0	0	0	1	0	0	0
10	C_1'	0	0	0	0	1	0	0	0
11	C_2	0	0	0	0	0	1	0	0
12	C_2'	0	0	0	0	0	1	0	0
13	C_3	0	0	0	0	0	0	1	0
14	C_3'	0	0	0	0	0	0	1	0
15	C_4	0	0	0	0	0	0	0	1
16	C_4'	0	0	0	0	0	0	0	1
	Goal	1	1	1	1	3	3	3	3

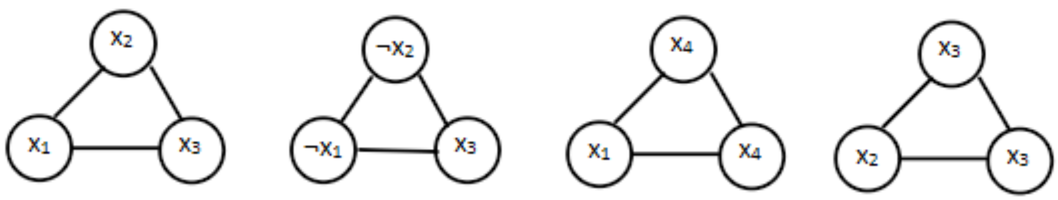
To achieve the desired sum, below rows can be selected: 1 (x_1=True), 3 (x_2=True), 5 (x_3=True), 7 (x_4=True), 11 (C_2=True), and 12 (C_2'=True). In this case, all clauses are satisfied.

(2) The equivalent Vertex Cover instance:

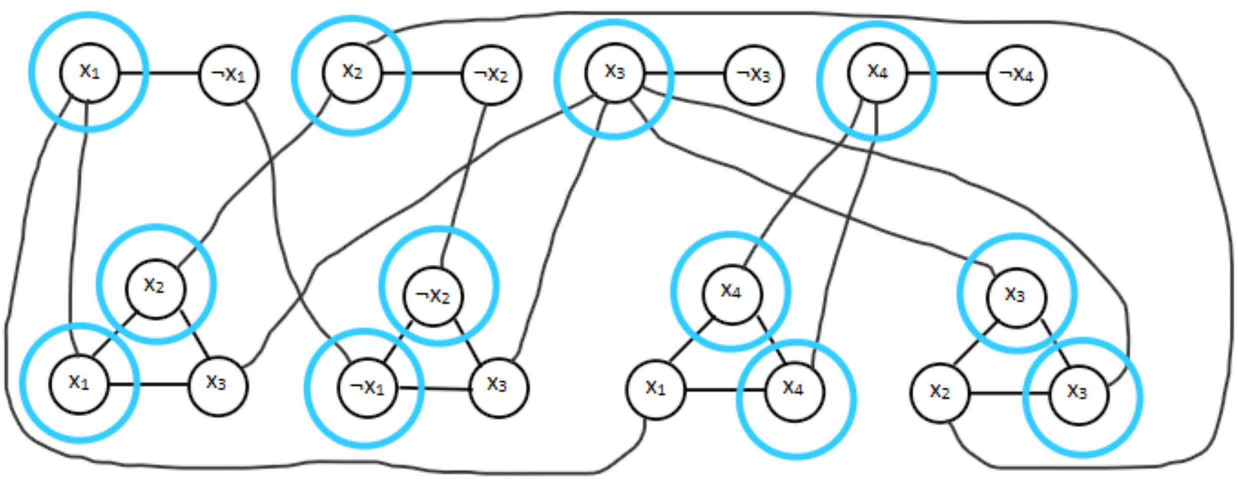
Variable gadgets:



Clause gadgets:



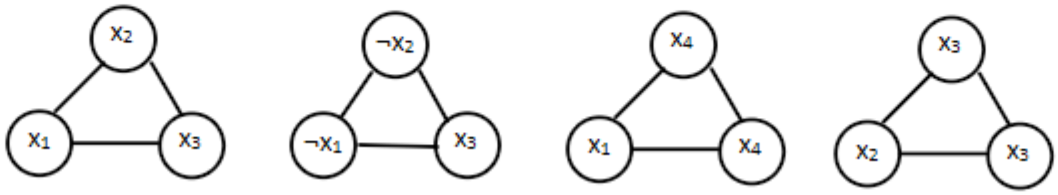
Combined gadgets:



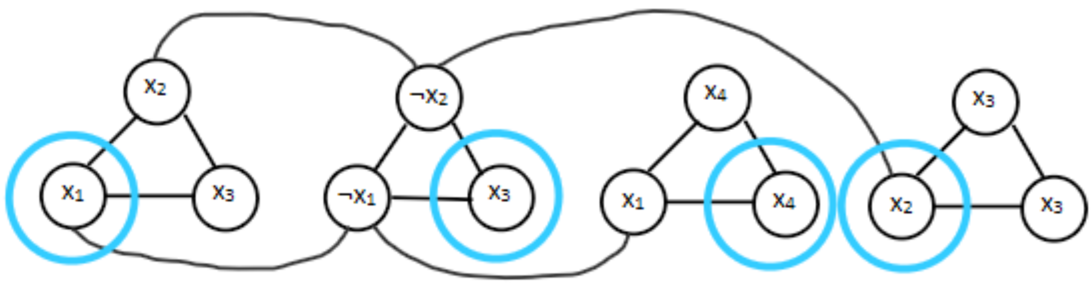
The number of vertices needed to be selected is k = n + 2m = 4 (the number of variables) + 2 x 4 (the number of clauses) = 12. Since the graph above has a vertex cover with exact 12 vertices (the circled ones), all clauses are satisfied.

(3) The equivalent Independent Set instance:

Clause gadgets:



Combined gadgets:

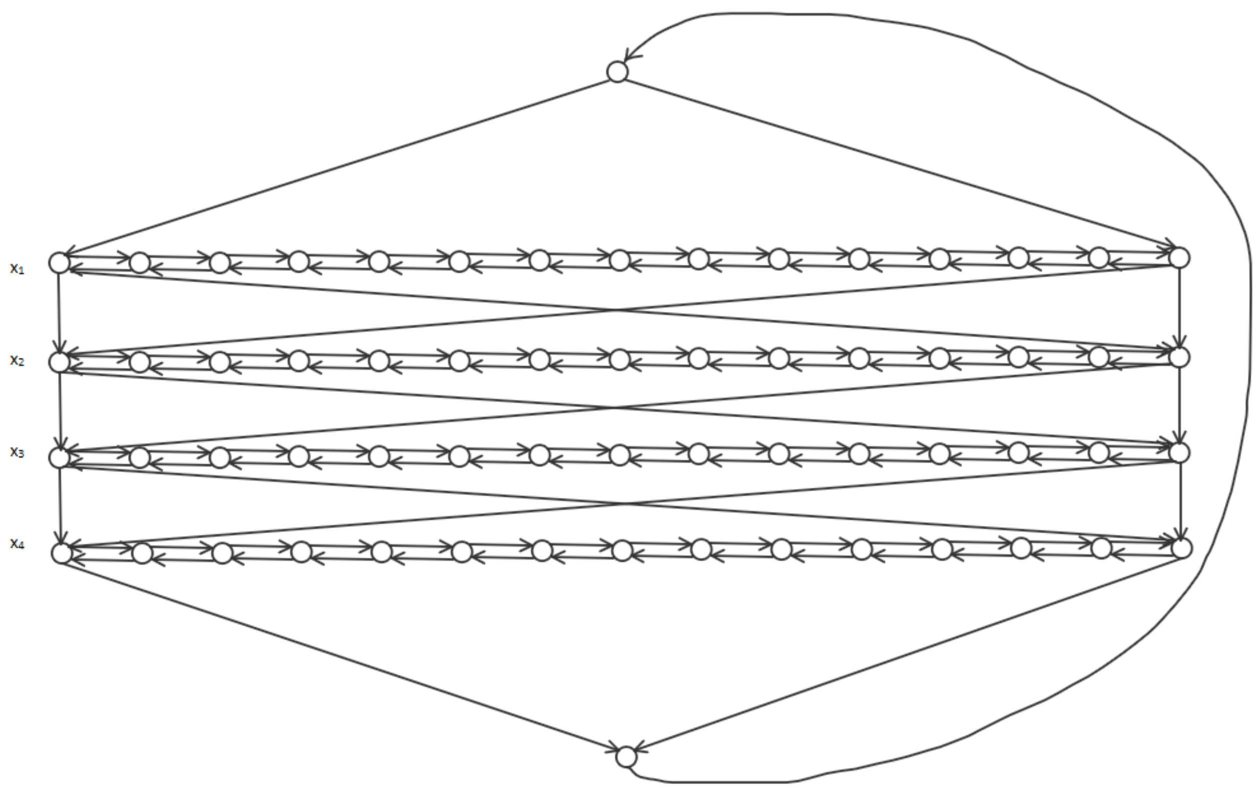


The number of vertices needed to be selected in the independent set is k = m = 4 (the number of clauses). Since the graph above has an independent set with exact 4 vertices (the circled ones), all clauses are satisfied.

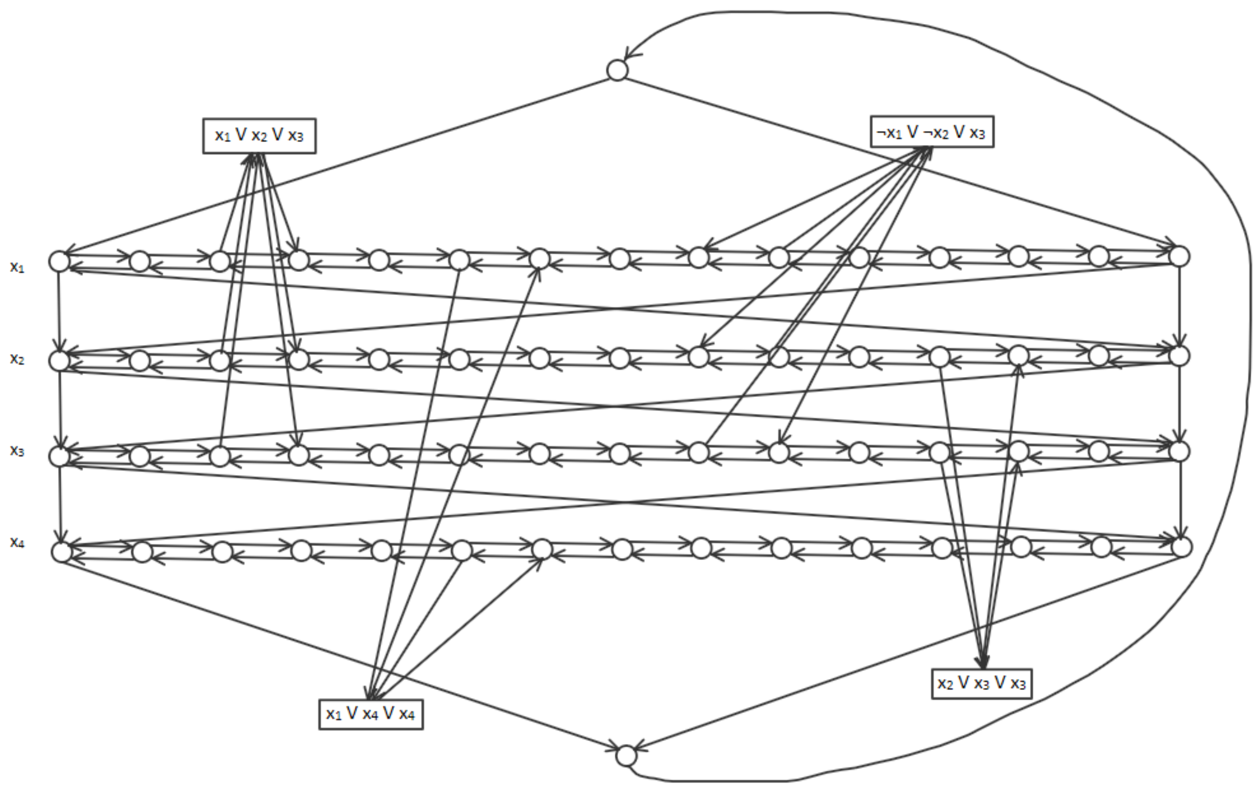
(4) The equivalent Hamiltonian Circuit instance:

Assume for each path i has 3m + 3 vertices (i.e. vertex 1, vertex 2, …, vertex 3m + 3), where m is the number of clauses. If variable x_i is True, the direction of passing the path i is left to right. If variable $\neg x_i$ is True, the direction of passing the path i is right to left. For clause C_j, if x_i is in C_j, C_j, has edge from vertex 3j to vertex 3j + 1; if $\neg x_i$ is in C_j, C_j, has edge from vertex 3j + 1 to vertex 3j.

Variable gadgets:



Combined gadgets:



Below is the graph with a Hamiltonian Circuit highlighted, indicating all clauses are satisfied: