Assignment#4 Key

1. NotDominating(ND) = { f | for some x, f(x) < x }.

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **ND**.

∃<x,t> [STP(f,x,t) & (VALUE(f,x,t) < x)]}

b.) Use Rice's Theorem to prove that **ND** is undecidable. Be Complete.

ND is non-trivial as **C0(x) = 0** ∈ **ND** and **S(x) = x+1** ∉ **ND**

Let f,g be two arbitrary indices of procedures such that ∀**x f(x) = g(x)**

f ∈ **ND** iff **∃x f(x) < x iff** $f(x_0) < x_0$ **for some** x_0 **iff** $g(x_0) < x_0$ **as** ∀**x f(x) = g(x) implies ∃x g(x) < x iff g** ∈ **ND**

f ∉ **ND** iff ∀**x [f(x)↓ implies f(x)>x] iff** ∀**x [g(x)↓ implies g(x)>x] as** ∀**x f(x) = g(x) iff g** ∉ **ND**

c.) Show that **K = { f | f(f)** converges **}** is many-one reducible to **ND**.

Let **f** be an arbitrary index. From **f**, define ∀**x** $F_f(x)$ **= f(f)-f(f)**.
f ∈ **K** implies ∀**x** $F_f(x) = 0$ implies F_f ∈ **ND**.
f ∉ **K** implies ∀**x** $F_f(x)$ diverges implies F_f ∉ **ND**.

Thus, **K** $\leq_m$ **ND**

d.) Show that **ND** is many-one reducible to **K = { f | f(f)** converges **}**

Let **f** be an arbitrary index. From **f**, define ∀**y** $F_f(y)$ **= ∃<x,t> STP(f,x,t) & (VALUE(f,x,t) < x)**
f ∈ **ND** implies ∀**y** $F_f(y)$ converges implies $F_f(F_f)$ converges implies F_f ∈ **K**
f ∉ **ND** implies ∀**y** $F_f(y)$ diverges implies F_f ∉ **K**.

Thus, **ND** $\leq_m$ **K**

2. AlwaysDominates(AD) = { f | for all x, f(x) > x }

a.) Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of **AD**.

∀x ∃t [STP(f,x,t) & (VALUE(f,x,t) > x)]}

b.) Use Rice's Theorem to prove that **AD** is undecidable. Be Complete.

AD is non-trivial as **S(x) = x+1** ∈ **AD** and **C0(x) = 0** ∉ **AD**

Let f,g be two arbitrary indices of procedures such that ∀**x f(x) = g(x)**

f ∈ **AD** iff ∀**x f(x) < x iff** ∀**x g(x) < x iff g** ∈ **ND**

c.) Show that **TOT = { f |** for all **x**, f**(x)** converges **}** is many-one reducible to **AD**.

Let **f** be an arbitrary index. From **f**, define ∀**x** $F_f(x) = f(x)-f(x) + x + 1$.
f ∈ **TOT** implies ∀**x** $F_f(x) = x+1$ implies F_f ∈ **AD**.
f ∉ **TOT** implies ∃**x** $F_f(x)$ diverges implies F_f ∉ **AD**.

Thus, **TOT** $\leq_m$ **AD**

d.) Show that **AD** is many-one reducible to **TOT = { f |** for all **x**, f**(x)** converges **}**

Let **f** be an arbitrary index. From **f**, define ∀**x** $F_f(x) = \mu y\ [\ f(x) > x\ ]$
f ∈ **AD** implies ∀**x** $F_f(x)$ converges implies F_f ∈ **TOT**
f ∉ **AD** implies ∃**x** $F_f(x)$ diverges implies F_f ∉ **TOT**.

Thus, **AD** $\leq_m$ **TOT**