Assignment#3 Key

1. Check F to be sure it's legit

- **GreaterThanZero(F, 0) = exp(F, 0) > 0**
  GreaterThanZero(F, y+1) = GreaterThanZero(F,y) && (exp(F, y+1) > 0)
- **N(F) = exp(F, 0)**
- **WellFormed(F) = GreaterThanZero(F, 2*N(F)) &&**
  (exp(F, 2*N(F)+1) == 1) && (exp(F, 2*N(F)+2) == 1)
- Really, really inefficient
  NoExtraPrimeFactors(F) = ~∃ 2*(N(F)+1)<z≤F//2 [exp(F, z) > 0]
- **AllGood(F) = WellFormed(F) && NoExtraPrimeFactors(F)**
- There are other approaches for **NoExtraPrimeFactors**

1. Alternate approach to **NoExtraPrimeFactors**

- **ValueOf_F(F, 0) = prime(0) ^ exp(F, 0)**
  ValueOf_F(F, y+1) = ValueOf_F(F, y) * (prime(y+1) ^ exp(F, y+1))
- **ExpectedValueOf_F(F) = ValueOf_F(F, exp(F, 2*N(F)+1)**
- **NoExtraPrimeFactors(F) = ExpectedValueOf_F(F) == F**
- The idea here is that we compute the expected value of **F** from **F**, by multiplying all its expected factors. If there is an extraneous factor (a prime factor **>** $\mathbf{p_{2n+2}}$) then our actual value of **F** will be larger than expected and that can be checked just by using equality or, if you prefer, **ExpectedValueOf_F(F) < F**

2. Show S inf. rec. iff S is the range of a monotonically increasing function

- Let $\mathbf{f_S(x+1) > f_S(x)}$, and $\mathbf{Range(f_S(x)) = S}$. **S** is decided by the characteristic function
  $\chi_S(x) = \exists\ y \le x\ [\ \mathbf{f_S(y) == x}\ ]$
  The above works as **x** must show up within the first **x+1** numbers listed since $\mathbf{f_S}$ is monotonically increasing.
- Let **S** be infinite recursive. As **S** is recursive, it has a characteristic function where $\chi_S(x)$ is true iff **x** is in **S**.
  Define the monotonically increasing enumerating function $\mathbf{f_S(x)}$ where
  $\mathbf{f_S(0) = \mu\ x\ [}\ \chi_S(x)\ ]$
  $\mathbf{f_S(y+1) = \mu\ x{>}f_S(y)\ [}\ \chi_S(x)\ ]$
  As required, this enumerates the elements of **S** in order, low to high.

3. If S is infinite re, then S has an infinite recursive subset R

- Let $\mathbf{f_S}$ be an algorithm where $\mathbf{S = range(f_S)}$ is an infinite set
- Define the monotonically increasing function $\mathbf{f_R(x)}$ by
  $\mathbf{f_R(0) = f_S(0)}$
  $\mathbf{f_R(y+1) = f_S(\ \mu\ x\ [\ f_S(x) > f_R(y)\ ]\ )}$
- The above is monotonically increasing because each iteration seeks a larger number and it will always succeed since **S** is itself infinite and so has no largest value. Also, **R** is clearly a subset of **S** since each element is in the range of $\mathbf{f_S}$.
- From #2, **R** is infinite recursive as it is the range of a monotonically increasing algorithm $\mathbf{f_R}$.
- Combining, **R** is an infinite recursive subset of **S**, as was desired.