Assignment #5; Due February 26 at start of class

1. Consider the set of indices $\text{SemiConstant} = SC = \{ f \mid |\text{range}(\varphi_f)| = 1 \}$.

   a) Using $\text{STP}$, $\text{VALUE}$ and a minimum number of alternating quantifiers, describe the set $\text{SemiConstant}$.
      $$\exists <x,t> \forall <y,s> [\text{STP}(f,x,t) \land (\text{STP}(f,y,s) \implies \text{VALUE}(f,x,t) = \text{VALUE}(f,y,s))]$$

   b) Show that $K \leq_m \text{SemiConstant}$, where $K = \{ f \mid \varphi_f(f) \downarrow \}$.
      Let $f$ be arbitrary. Define an algorithmic mapping $G$ from indices to indices as $G_f(x) = f(f)$. Now, the range of $G_f = \{f(f)\}$. If $f$ is in $K$, then this range is a singleton value and so $G_f$ is in $SC$. If $f$ is not in $K$, then this range is empty and so $G_f$ is not in $SC$. Thus, $K \leq_m SC$.

c) Use Rice’s Theorem to show that $\text{SemiConstant}$ is not recursive (not decidable).
   Note that members of $\text{SemiConstant}$ do not need to converge for all input, but they must converge on at least one input and when they do converge they always produce the same output value. Hint: There are two properties that must be demonstrated.

   First, $SC$ is non-trivial as $Z(x) = 0$ is in $SC$ and $Z(x) = x$ is not.

   Second, $SC$ is an I/O Property.
   To see this, let $f$ and $g$ be arbitrary indices of computable functions such that
   $$\forall x \varphi_f(x) = \varphi_g(x).$$
   $f$ is in $SC$ iff $|\text{range}(\varphi_f)| = 1$. But $g$’s range is exactly that of $f$ and so,
   $|\text{range}(\varphi_f)| = 1$ iff $|\text{range}(\varphi_g)| = 1$. But then,
   $f$ is in $SC$ iff $g$ is in $SC$

   Since $SC$ is not trivial and is an I/O property then it is not recursive by Rice’s Theorem.