Assignment #4; Due February 24 at start of class

Choosing from among (REC) recursive, (RE) re non-recursive, (coRE) co-re non-recursive, (NRNC) non-re/non-co-re, categorize each of the sets in a) through d). Justify your answer by showing some minimal quantification of some known recursive predicate.

a.) \[ \{ < f, g > | \text{domain}(\varphi_f) \subseteq \text{domain}(\varphi_g) \} \]

\[ \text{Justification:} \]
Note: This allows equal domains, but even works if domain(\varphi_f) is \( \emptyset \) and domain(\varphi_g) is \( \mathbb{N} \).

b.) \[ \{ f | \text{no number appears more than once in range}(\varphi_f) \} \]

\[ \text{Justification:} \]
Note: This can include functions whose ranges are empty and even those whose ranges do include all natural numbers.

c.) \[ \{ f | \varphi_f(f) \downarrow \text{in at most } f+1 \text{ steps} \} \]

\[ \text{Justification:} \]
Note: This is similar to the set \( K \) but involves an added twist.

d.) \[ \{ f | \varphi_f(f) \downarrow \text{but takes at least } f+1 \text{ steps to do so} \} \]

\[ \text{Justification:} \]
Note: This is also similar to \( K \) but has a twist that differs from that in part (c).

e.) \[ \{ < f, x, y > | \varphi_f(x) \downarrow \text{ and } \varphi_f(y) \downarrow \text{ but } \varphi_f(x) \text{ takes longer to converge than does } \varphi_f(y) \} \]

\[ \text{Justification:} \]
Note: Be careful to address the fact that \( \varphi_f \) converges on both \( x \) and \( y \).