Name: PID:

# COT5405 – Bonus Homework

Out date: **11/29/2010 (Monday)**, due date: **12/08/2010 (Wednesday)**

[15 points] Consider the knapsack problem without repetition. We studied a DP-algorithm for this problem with running time *O(nW)*, where n is the number of items and W is the capacity of the knapsack.

Find a DP-algorithm with running time *O(nV)*, where $V=v_1+v_2+\ldots+v_n$ is the sum of the values.

a) Give and define the state of your recurrence.
b) Define the base case(s) of your state.
c) Give the recurrence.
d) State the output.
e) State the memory requirement. Give arguments why the running time is *O(nV)* and why the memory requirement is equal to the value you have determined.

**Answers:**

a)

b)

c)

d)

e)

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[15 points] Consider the approximation algorithm for vertex cover with approximation ratio 2.

a) What is the optimal vertex cover of the complete graph $K_n$ on $n$ vertices, i.e., its edges are *(1,2), (1,3),…,(1,n),…,(n-1,n).* What vertex cover does the approximation algorithm find if it looks at the edges in lexicographical order?
b) Same problem with the path graph $P_n$, i.e., its edges are *(1,2),(2,3),…,(n-1,n).*
c) Same problem with the circle graph $C_n$, i.e., its edges are *(1,2),(2,3),…(n-1,n),(1,n).*
d) Same problem with the binary tree graph $T_n$, where $n$ is a power of 2. Assume the root of tree is located at the top and the leaves at the bottom. In this subproblem, the edges are processed from left to right and bottom to top.

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[20 points] Show that the problem of finding the minimum vertex cover in a bipartite graph reduces to maximum flow. Describe the reduction in a precise and concise way (no lengthy paragraphs of text).