**Fractional Knapsack Problem (fKSP)**

**Given:** a set of n objects {1, 2, …, n}

 A set of profits {p1, p2, …, pn}

 A set of n weights {w1, w2, …, wn}

 A total weight W.

**Assumption:** Object i has profit pi and weight wi, for 1 ≤ i ≤ n. We may place in the knapsack αi percent of object i and receive αipi profit at a weight of αiwi, where 0 ≤ αi ≤ 1.

**Goal:** Find a solution vector A = (α1, α2, …, αn) so that

 1) Σαi wi ≤ W, and

 2) Σαi pi is maximum.

**Proposed solution:**

 {We assume Σwi ≤ W, otherwise the solution is to set each αi to 1.}

 (1) Relabel and sort the objects so that

p1/w1 ≥ p2/w2 ≥ … ≥ pn/wn. {This requires O(nlog2n) time.}

(2) Set W' 🡨 0 and k 🡨 1

(3) While W'+wk < W do

W' 🡨 W'+wk, αk 🡨 1, k 🡨 k+1

(4) αk 🡨 (W–W')/wk

(5) for i = k+1 to n set αi 🡨 0

**Conjecture:**

 1) Σαi wi ≤ W, and

 2) Σαi pi is maximum.

**Assignment:** Prove the conjecture is true.

 **Hint 1:** Assume it is not true. Then, there exists a vector B = (β1, β2, …, βn) where

(1) 0 ≤ β1, ≤ 1 for 1 ≤ i ≤ n,

(2) Σαi wi ≤ W, and

(3) Σβipi  > Σαi pi

**Hint 2:** There may be several such vectors, so assume you have one for which there is a largest index j such that βi = 1, for 1 ≤ i ≤ j, and βj+1 < 1. Note that it is possible that j = 0 (That just means β1 < 1 for all "better solutions").

**An example of a fKSP where truncation doesn't work (well, anyway).**

W = 100

profit = 66 20 30 60 40

weight = 30 10 20 50 40

p/w = 2.2 2.0 1.5 1.2 1.0

The solution for fKSP is (1, 1, 1, .8, 0) and gives a value of 164.

By truncating the fractional item, we get 116.

The optimal 0-1 KSP solution is (1, 1, 1, 0, 1) giving 156.