COT 5310 Fall 2007 Midterm#1 Name: KEY

12 **1**. Choosing from among **(REC) recursive**, **(RE) re non-recursive, (coRE) co-re non-recursive**, **(NR) non-re/non-co-re**, categorize each of the sets in a) through d). Justify your answer by showing some minimal quantification of some known recursive predicate.

a.) { f | whenever x>y and f(x)↓ and f(y)↓ then f(x)>f(y) } ***coRE***

Justification: ***∀x ∀y ∀t [(STP(x,f,t) & STP(y,f,t) & (x>y))⇒(VALUE(x,f,t)>VALUE(y,f,t))]***

b.) { f | size of range(f) is at most 1 } ***coRE***

Justification: ***∀x ∀y ∀t [(STP(x, f, t) & STP(y, f, t)) ⇒ (VALUE(x, f, t) = VALUE(y, f, t))]***

I allowed ∃K ∀x ∀t [STP(x, f, t) ⇒ (VALUE(x, f, t) = K)], which is NR

c.) { <f,x> | f(x) converges in at most x^2 steps } ***REC***

Justification: ***STP(x, f, x^2)***

d.) { f | domain(f) contains the value 0 } ***RE***

Justification: ***∃t STP(0, f, t)***

12 **2**. Let set **A** be recursive, and both **B** and **C** be re non-recursive. Choosing from among **(REC) recursive**, **(RE) re non-recursive**, **(NR) non-re**, categorize the set **D** in each of a) through d) by listing **all** possible categories. No justification is required. You may find it useful to know that, if **E** is recursive (re, non-re) then so is $\mathbf{E_{even} = \{ 2x \mid x \in E\}}$ and $\mathbf{E_{odd} = \{2x+1 \mid x \in E\}}$.

a.) D ⊇ B ***REC, RE, NR***

b.) D = ~A ***REC***

c.) D = B ∪ C ***REC, RE***

8 **3.** Let **S** be an arbitrary non-empty re set. Furthermore, let **S** be the range of some partial recursive function $\mathbf{f_s}$. Show that **S** is the range of some primitive recursive function, call it $\mathbf{h_s}$.

Let $h_s(<x,t>) = a * (1 - STP(x, f_s , t)) + VALUE(x, f_s , t) * STP(x, f_s , t)$

where 'a' is some arbitrary element of S. Such an 'a' exists since S is non-empty.

First, h_s is primitive since the constants C_a and C_1, STP, VALUE, +, * and – are all primitive recursive and the primitive recursive functions are closed under composition.

Now, given any element y of S, there must exist some x such that $f_s(x) = y$. For such an x, there must exist a t such that STP(x, f_s , t) and VALUE(x, f_s , t) = y . Of course, if such an <x,t> exists, there are an infinite number of these (<x,t'> for t' > t). Clearly, for each such <x,t>, $h_s(<x,t>) = y$. If, on the other hand, ~STP(x, f_s , t), then $h_s(<x,t>) = a$. Thus, all and only the elements in S are enumerated by h_s, exactly what we need.

8 **4.** Prove that the **Halting Problem** (the set $\mathbf{HALT = K_0 = L_u}$) is not recursive (decidable) within any formal model of computation. (Hint: A diagonalization proof is required here.)

Look at notes.

8 **5.** Using reduction from the known undecidable set **HasZero, HZ = { f | ∃x f(x) = 0 }**, show the non-recursiveness (undecidability) of the problem to decide if an arbitrary partial recursive function is in the set **Identity, ID = { f | ∀x f(x) = x }**.

HZ = { f | ∃x ∃t [STP(x, f, t) & VALUE(x, f, t) == 0] }

Let f be the index of an arbitrary effective procedure.

Define $g_f(y)$ = y * ∃x ∃t [STP(x, f, t) & VALUE(x, f, t) == 0]

If ∃x f(x) = 0, we will find an x and a run-time t, and so we will return y (y * 1)

If ∀x f(x) ≠ 0, then we will diverge in the search process and never return a value.

Thus, f ∈ HZ iff g_f ∈ ID.

7 **6.** Assuming only the primitive recursiveness of $\mathbf{C_a(x_1,\dots,x_n) = a}$: constants; $\mathbf{I_i^n (x_1,\dots,x_n) = x_i}$: projection; and **S(x) = x+1** : increment; and that the prf's are closed under composition and primitive recursion (iteration), show the primitive recursiveness of the function **Max(x , y, z)**.

pred(0) = 0, pred(y+1) = y; -(x,0) = x, -(x,y+1) = pred(-(x,y)); signum(0) = 0, signum(y+1) = 1; >(x,y) =signum(-(x,y)); +(x,0) = x, +(x,y+1) = S(+(x,y)); Max2(x,y) = x* >(x,y) + y * >(y,x)

7 **7**. Demonstrate a **Register Machine** where the numbers **x**, **y** and **z** are in registers **R2**, **R3** and **R4**, respectively, all other registers being zero. **Max(x, y, z)** ends up in register **R1**. The final contents of all registers except **R1** are unimportant.

1. DEC2[2,5]
2. DEC3[3,3]
3. DEC4[4,4]
4. INC1[1]
5. DEC3[6,8]
6. DEC4[7,7]
7. INC1[5]
8. DEC4[9,10]
9. INC1[8]
10.

5 **8**. Present an ordered **Factor Replacement System** (ordered rules of form **ax → bx**) that, when started on the number $3^x 5^y 7^z$, terminates on the number $2^{Max(x,y,z)}$. When you end, no prime factor other than perhaps **2** should appear in the final number.

3 5 7 x → 2 x
3 5 x → 2 x
3 7 x → 2 x
5 7 x → 2 x
3 x → 2 x
5 x → 2 x
7 x → 2 x